Math Connections to Earth and Space Science

Glen Schuster

Endeavor

Math Connections to Earth and Space Science

Mathematical Problems in Earth and Space Science

Dr. Sten Odenwald
NASA

http://spacemath.gsfc.nasa.gov

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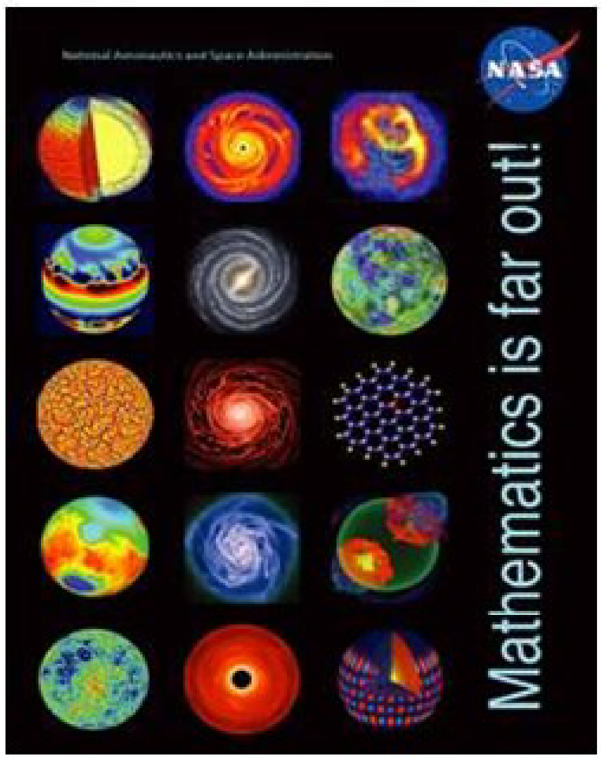
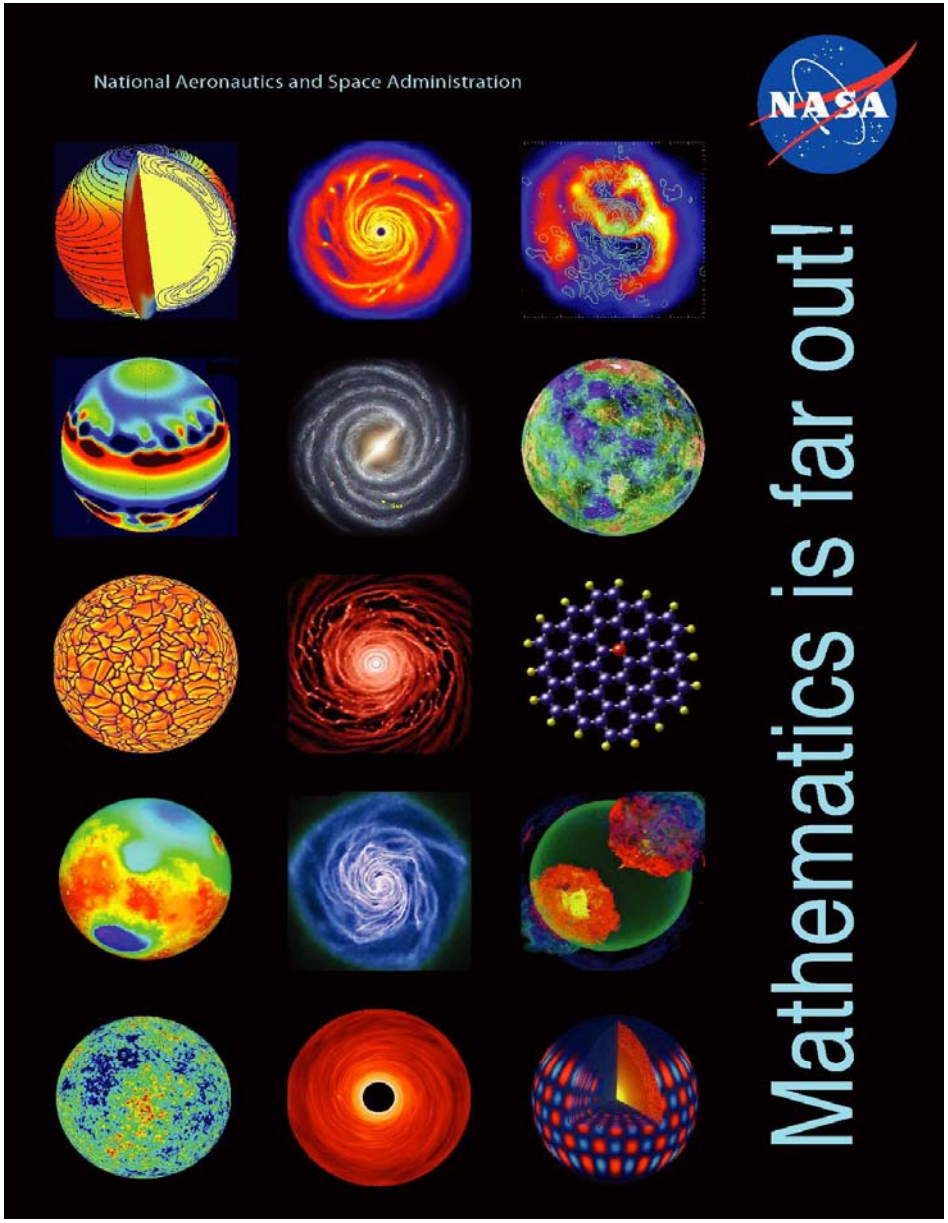
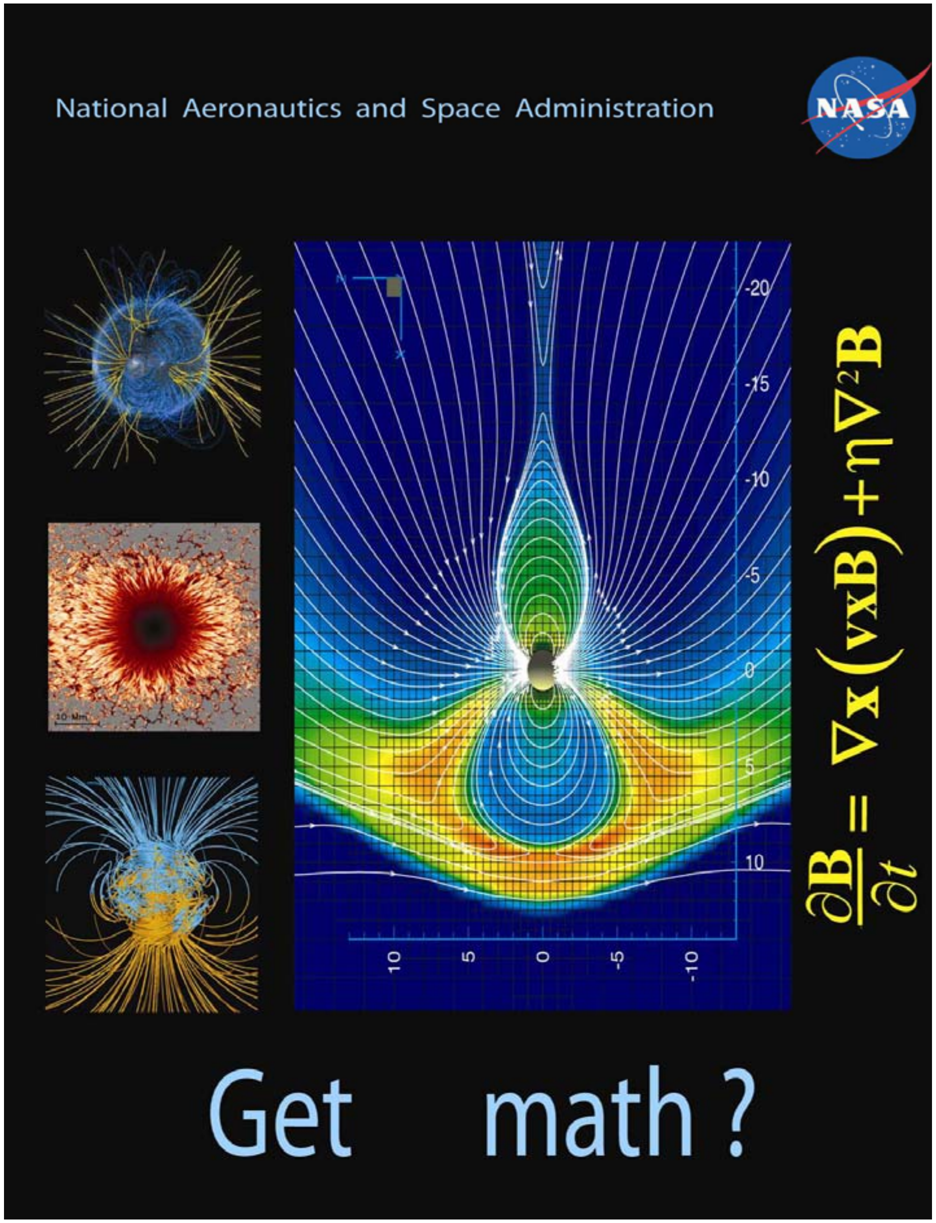


This web page contains problem sets in PDF format. The goal of these problems is to teach students about astronomy and space science by using mathematics and real-world problems. The current batch is for January-April, 2010. As new NASA press releases are posted, a small number of additional math problems will appear. For email announcements about Space Math, join the E-Teachers Listserve.

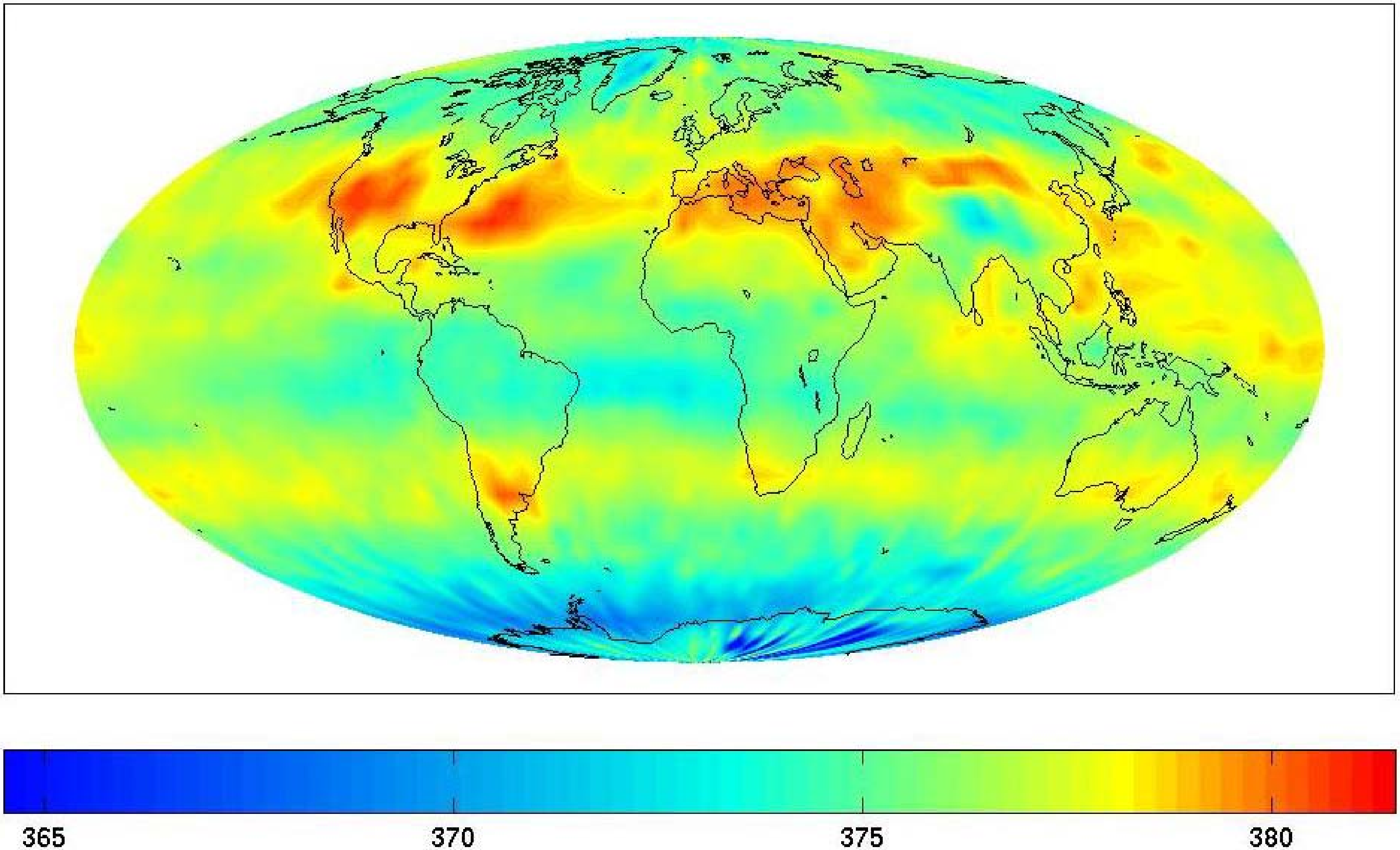
http://spacemath.gsfc.nasa.gov

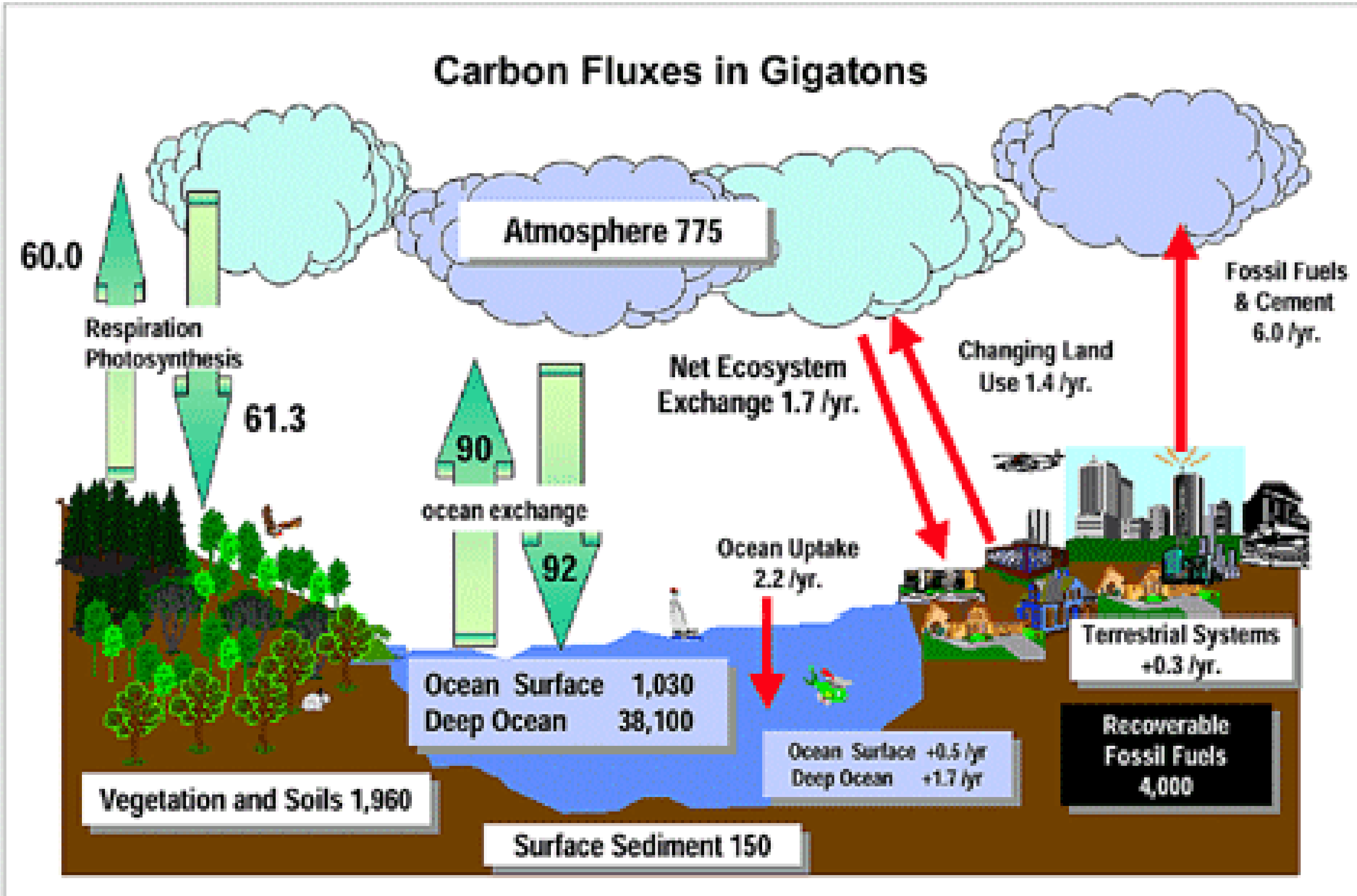


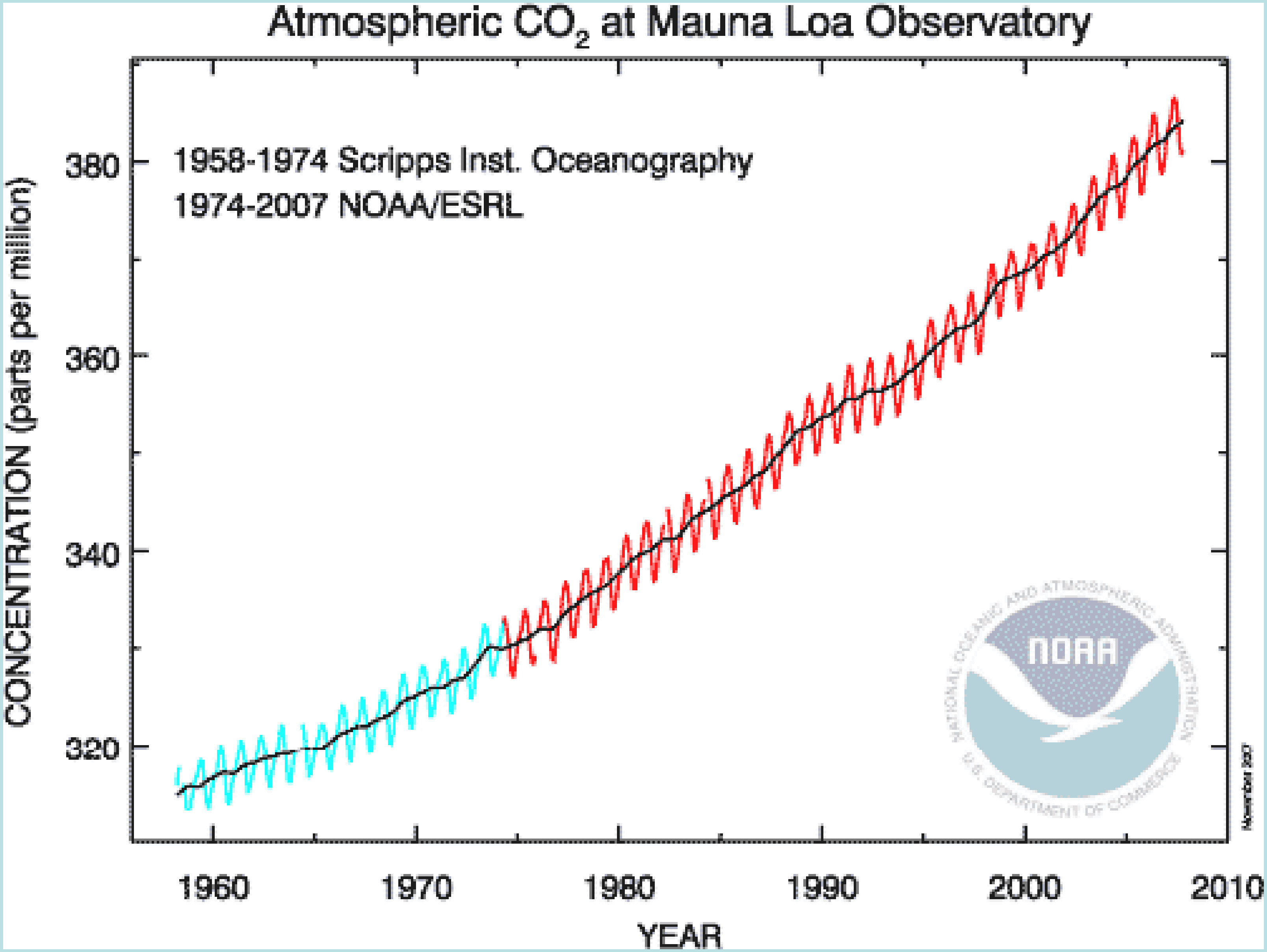



CO2

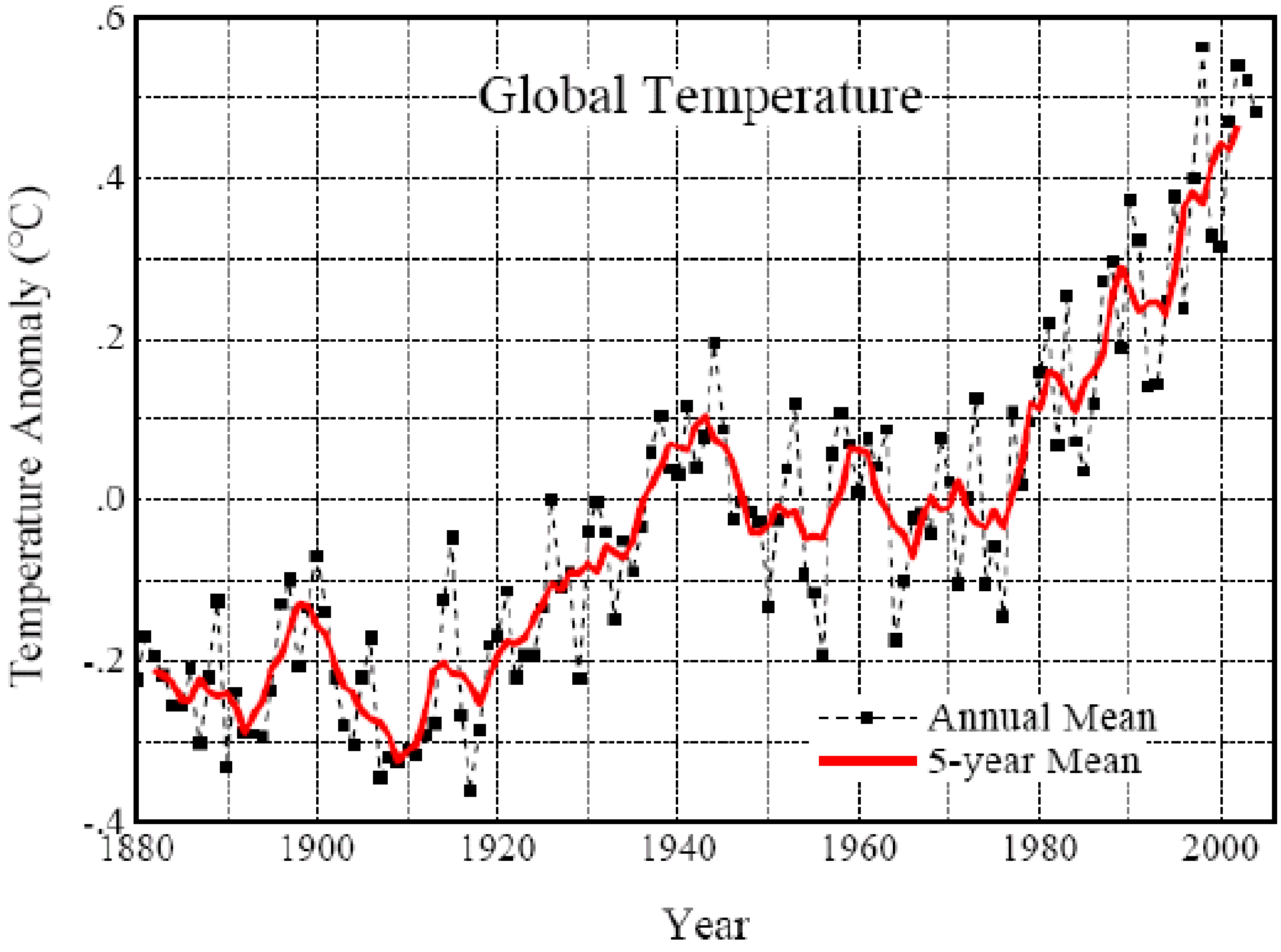

I
CO2
ilovecarbondioxide.com

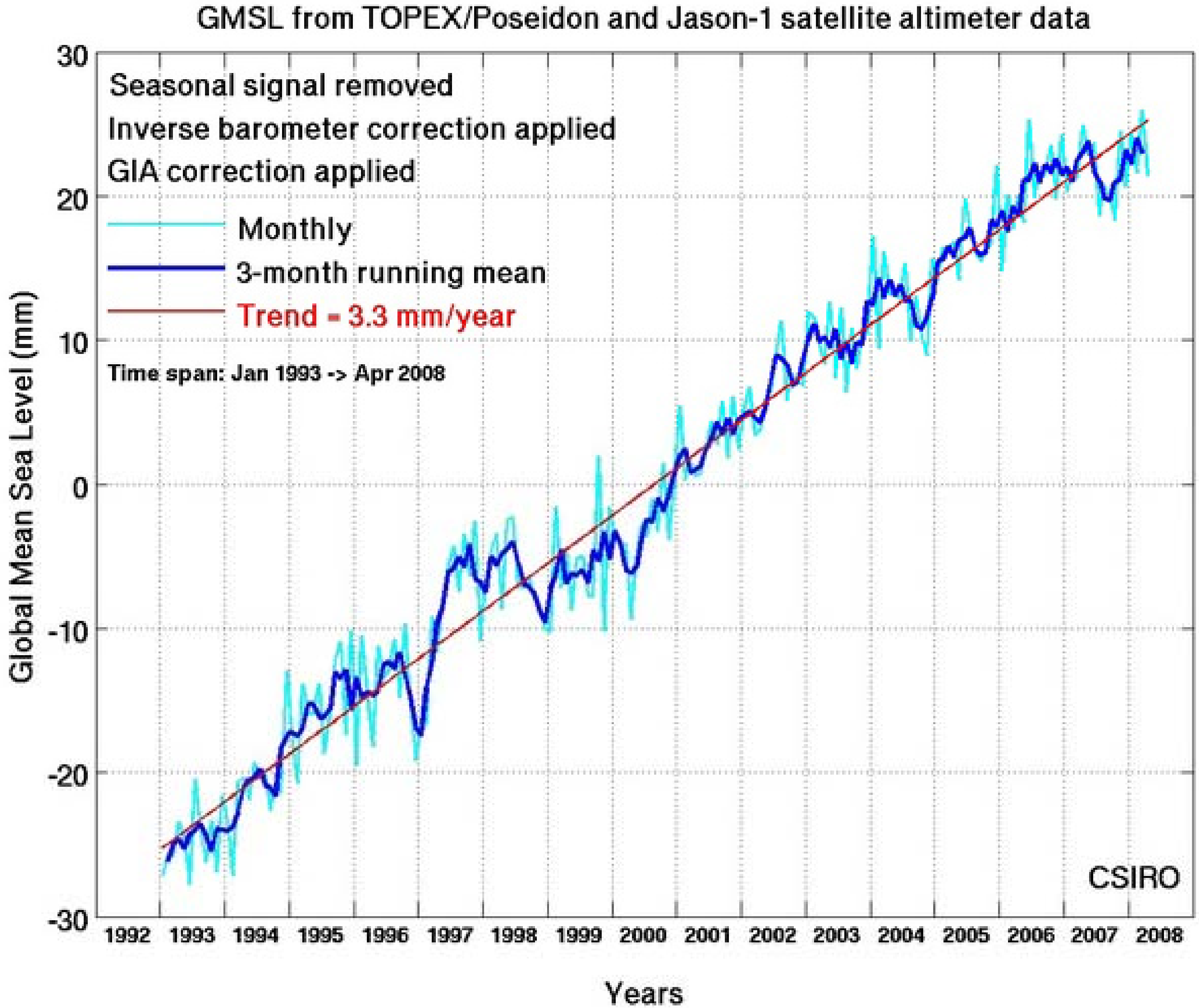

365
370
375
380


Carbon Fluxes in Gigatons
Atmosphere 775
60.0
Respiration
Photosynthesis
61.3
90
ocean exchange
92
Net Ecosystem
Exchange 1.7 /yr.
Changing Land
Use 1.4 /yr.
Fossil Fuels
& Cement
6.0 /yr.
Ocean Uptake
2.2 /yr.
Ocean Surface 1,030
Deep Ocean 38,100
Ocean Surface +0.5 /yr
Deep Ocean +1.7 /yr
Terrestrial Systems
+0.3 /yr.
Recoverable
Fossil Fuels
4,000
Vegetation and Soils 1,960
Surface Sediment 150



Atmospheric CO_2 at Mauna Loa Observatory
1958-1974 Scripps Inst. Oceanography
1974-2007 NOAA/ESRL
CONCENTRATION (parts per million)
380
360
340
320
1960
1970
1980
1990
2000
2010
YEAR
NOAA
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
U.S. DEPARTMENT OF COMMERCE


Global Temperature
Temperature Anomaly (°C)
.6
.4
.2
.0
-.2
-.4
1880
1900
1920
1940
1960
1980
2000
Year
Annual Mean
5-year Mean


GMSL from TOPEX/Poseidon and Jason-1 satellite altimeter data
Seasonal signal removed
Inverse barometer correction applied
GIA correction applied
Monthly
3-month running mean
Trend = 3.3 mm/year
Time span: Jan 1993 -> Apr 2008
Global Mean Sea Level (mm)
30
20
10
0
-10
-20
-30
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
Years
CSIRO


GE Watthour Meter
BO-140947
15 935 007


soft white 60
regular, everyday light

Goal:

Convert your annual electric bill into an equivalent number of tons of CO_2

Concepts and Definitions:

Watt - Unit of electrical energy use

$$1 \text{ watt} = \frac{1 \text{ Joule of energy}}{1 \text{ second}}$$

Concepts and Definitions:

kiloWatt - 1,000 watts

‘kilo’ is the prefix for 1,000

Examples:

Concepts and Definitions:

kiloWatt - 1,000 watts

'kilo' is the prefix for 1,000

Examples:

1 kilometer = 1,000 meters

1 kilogram = 1,000 grams

1 kiloton = 1,000 tons

Concepts and Definitions:

kiloWatt-Hour – a unit of electric energy use.

There are 2 things involved here.

1 – The unit of a kilowatt
2 – The idea that energy use is the product of a rate x time:

Energy = Power x time

Example:

10, 100-watt bulbs left on for 2 hours

Power = 10 x 100 = 1,000 watts = 1 kW

Energy = Power x time

= 1 kW x 2 hours

= 2 kiloWatt-Hours

= 2 kWh

Now you do it!

Problem:

A student is in the habit of leaving her laptop on all the time even after going to bed.

If the laptop uses 100 watts, and is left on 24-hours a day, and 365 days a year, how much electrical energy is used in a year, in kWh?

Problem:

A student is in the habit of leaving her laptop on all the time even after going to bed. If the laptop uses 100 Watts, and is left on 24-hours a day, and 365 days a year, how much electrical energy is used in a year, in kWh?

Answer:

$$E = 100 \text{ Watts} \times \frac{24 \text{ hours}}{1 \text{ } \cancel{\text{Day}}} \times \frac{365 \text{ } \cancel{\text{days}}}{1 \text{ year}}$$

E = 876,000 Watt Hours (in 1 year)

E = 876 kiloWatt Hours (in 1 year)

E = 876 kWh (in 1 year)

Home Electrical Energy Bill

January	830 kWh
February	820 kWh
March	840 kWh
April	770 kWh
May	750 kWh
June	1160 kWh
July	1330 kWh
August	1110 kWh
September	1040 kWh
October	740 kWh
November	650 kWh
December	1100 kWh

Home Electrical Energy Bill

January	830 kWh
February	820 kWh
March	840 kWh
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August	1110 kWh
September	1040 kWh
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December	1100 kWh

11,140 kWh

Converting total electricity consumption to tons of CO_2.

- Electricity produced by burning fossil fuels generates 0.7 kilograms of CO_2 per kWh.

Converting total electricity consumption to tons of CO2.

- Electricity produced by burning fossil fuels generates 0.7 kilograms of CO2 per kWh.
- Electricity produced by nuclear power generates 0 kilograms of CO2

Converting total electricity consumption to tons of CO_2.

- Electricity produced by burning fossil fuels generates 0.7 kilograms of CO_2 per kWh.
- Electricity produced by nuclear power generates 0 kilograms of CO_2
- Electricity generated by wind, solar or hydro generates 0 kilograms of CO_2

Converting total electricity consumption to tons of CO_2.

If: 1 kWh = 0.7 kilograms of CO_2

and: E = 11,140 kWh

Problem: How much CO_2 did this home produce in one year?

Converting total electricity consumption to tons of CO_2.

If: 1 kWh = 0.7 kilograms of CO_2
and: E = 11,140 kWh

Problem: How much CO_2 did this home produce in one year?

M = 11,140 kWh x 0.7 kg
= 7,798 kilograms of CO_2
= 7.8 tons of CO_2

…And now for something

completely different !

Exoplanets

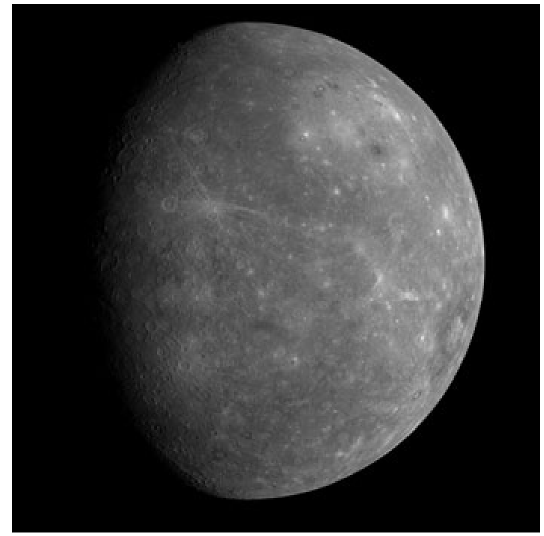
Habitable Zones

Ice
0 °Centigrade
+273 °Kelvin

Steam
+100 °Centigrade
+373 °Kelvin

Habitable Zones

Pluto - Ice
0 °Centigrade
+273 °Kelvin



Mercury - Steam
+100 °Centigrade
+373 °Kelvin

Habitable Zones:

A region of space surrounding a star where temperatures are suitable for water to remain in liquid form.

Habitable Zones:

A region of space surrounding a star where temperatures are suitable for water to remain in liquid form.

All living things require liquid water, hence this region is called a 'Habitable Zone'

Question:

What does the temperature of a planet depends on?

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What does the temperature of a planet depends on?

- √ Distance from star
- √ Brightness of star
- √ Reflectivity of planet
- √ Atmosphere of planet

Astronomers work with simple units:

Distance:

Sun to Earth = 150 million km

= 1 Astronomical Unit

= 1 AU

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Distance:
Sun to Earth = 150 million km
= 1 Astronomical Unit
= 1 AU

Luminosity of Sun = 4×10^{27} Watts
= 1 Solar Luminosity
= 1 Lsun

The temperature of a bare planet can be modeled by

$$T = 273 \frac{(1-A)^{1/4} \; L^{1/4}}{D^{1/2}}$$

A = Albedo

L = Luminosity of star

D = Distance to star

The temperature of a bare planet can be modeled by

$$T = 273 \sqrt[4]{\frac{(1-A)\ L}{D^2}}$$

A =Albedo L =Luminosity of star D =Distance to star

Example: Planet with albedo = 0.8
orbits at 1.5 AU around a star
with L = 5.0 Lsun

$$T = 273 \sqrt[4]{\frac{(1-0.8)\ 5.0}{1.5^2}}$$

$$T = 273\ (0.44)^{1/4}$$

T = 223 K (-58 F or -50 C)

Problem – What is the temperature for an earth-like planet for which D = 1.0 AU, L = 1.0 Lsun and A = 0.3 ?

$$T = 273 \sqrt[4]{\frac{(1-A)\ L}{D^2}}$$

A = Albedo

L = Luminosity of star

D = Distance to star

Problem – What is the temperature for an earth-like planet for which D = 1.0 AU, L = 1.0 Lsun and A = 0.3 ?

$$T = 273 \sqrt[4]{\frac{(1-0.3)\ \ 1}{1^{2}}}$$

T = 250 Kelvin (-23 C or -9 F)

Problem – How big is a Habitable Zone?

$$T = 273 \sqrt[4]{\frac{(1-A)\ L}{D^2}}$$

By assuming a value for A,
we can create the following table using MS Excel….

Distance from Star in AUs

LUMINOSITY

L	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
0.1	361	255	209	181	162	147	136	128	120	114	109	104	100	97	93	90
0.5	540	382	312	270	242	220	204	191	180	171	163	156	150	144	139	135
1	642	454	371	321	287	262	243	227	214	203	194	185	178	172	166	161
1.5	711	503	410	355	318	290	269	251	237	225	214	205	197	190	184	178
2	764	540	441	382	342	312	289	270	255	242	230	220	212	204	197	191
2.5	808	571	466	404	361	330	305	286	269	255	243	233	224	216	209	202
3	845	598	488	423	378	345	319	299	282	267	255	244	234	226	218	211
3.5	878	621	507	439	393	359	332	311	293	278	265	254	244	235	227	220
4	908	642	524	454	406	371	343	321	303	287	274	262	252	243	235	227
4.5	935	661	540	468	418	382	354	331	312	296	282	270	259	250	242	234
5	960	679	554	480	429	392	363	340	320	304	290	277	266	257	248	240
5.5	983	695	568	492	440	402	372	348	328	311	297	284	273	263	254	246
6	1005	711	580	503	450	410	380	355	335	318	303	290	279	269	260	251
6.5	1025	725	592	513	459	419	388	363	342	324	309	296	284	274	265	256
7	1045	739	603	522	467	426	395	369	348	330	315	302	290	279	270	261

Grid of planet temperatures

Problem: Where is 273 < T < 373 ?

Distance from Star in AUs

LUMINOSITY

L	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
0.1	361	255	209	181	162	147	136	128	120	114	109	104	100	97	93	90
0.5	540	382	312	270	242	220	204	191	180	171	163	156	150	144	139	135
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7	1045	739	603	522	467	426	395	369	348	330	315	302	290	279	270	261

Answer: See shaded region for approximate limits

Distance from Star in AUs

L	Area	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
0.1	0.12	361	255	209	181	162	147	136	128	120	114	109	104	100	97	93	90
0.5	0.48	540	382	312	270	242	220	204	191	180	171	163	156	150	144	139	135
1	1.08	642	454	371	321	287	262	243	227	214	203	194	185	178	172	166	161
1.5	1.32	711	503	410	355	318	290	269	251	237	225	214	205	197	190	184	178
2	1.92	764	540	441	382	342	312	289	270	255	242	230	220	212	204	197	191
2.5	2.24	808	571	466	404	361	330	305	286	269	255	243	233	224	216	209	202
3	3.00	845	598	488	423	378	345	319	299	282	267	255	244	234	226	218	211
3.5	3.40	878	621	507	439	393	359	332	311	293	278	265	254	244	235	227	220
4	3.40	908	642	524	454	406	371	343	321	303	287	274	262	252	243	235	227
4.5	2.88	935	661	540	468	418	382	354	331	312	296	282	270	259	250	242	234
5	3.80	960	679	554	480	429	392	363	340	320	304	290	277	266	257	248	240
5.5	4.80	983	695	568	492	440	402	372	348	328	311	297	284	273	263	254	246
6	5.28	1005	711	580	503	450	410	380	355	335	318	303	290	279	269	260	251
6.5	5.28	1025	725	592	513	459	419	388	363	342	324	309	296	284	274	265	256
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5	3.80	960	679	554	480	429	392	363	340	320	304	290	277	266	257	248	240
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7	6.44	1045	739	603	522	467	426	395	369	348	330	315	302	290	279	270	261

Problem: Red numbers give area of HZ. What pattern do you notice?

Answer: Stars more luminous that our sun (L=1) have much larger HZs

Distance from Star in AUs

L	Area	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	3.2
0.1	0.12	361	255	209	181	162	147	136	128	120	114	109	104	100	97	93	90
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430 planets known…and the search continues!