Math Connections to Earth and Space Science

Glen Schuster

Endeavor

NASA

"Mathematics of Sound: The Architecture of Acoustics"

Back Row (left to Right): Xavier Maciel, Matthew Sarrico, Carlos Viera, Freddy Pastrano, Bruno Coimbra, Mr. Henry Varum
Front Row (Left to Right): Nichole Bermudez, Hallana Trinetta, Maria Silva, Daniella Dias, Kelly Amorin

"Mathematics of Sound: "The Architecture of Acoustics"

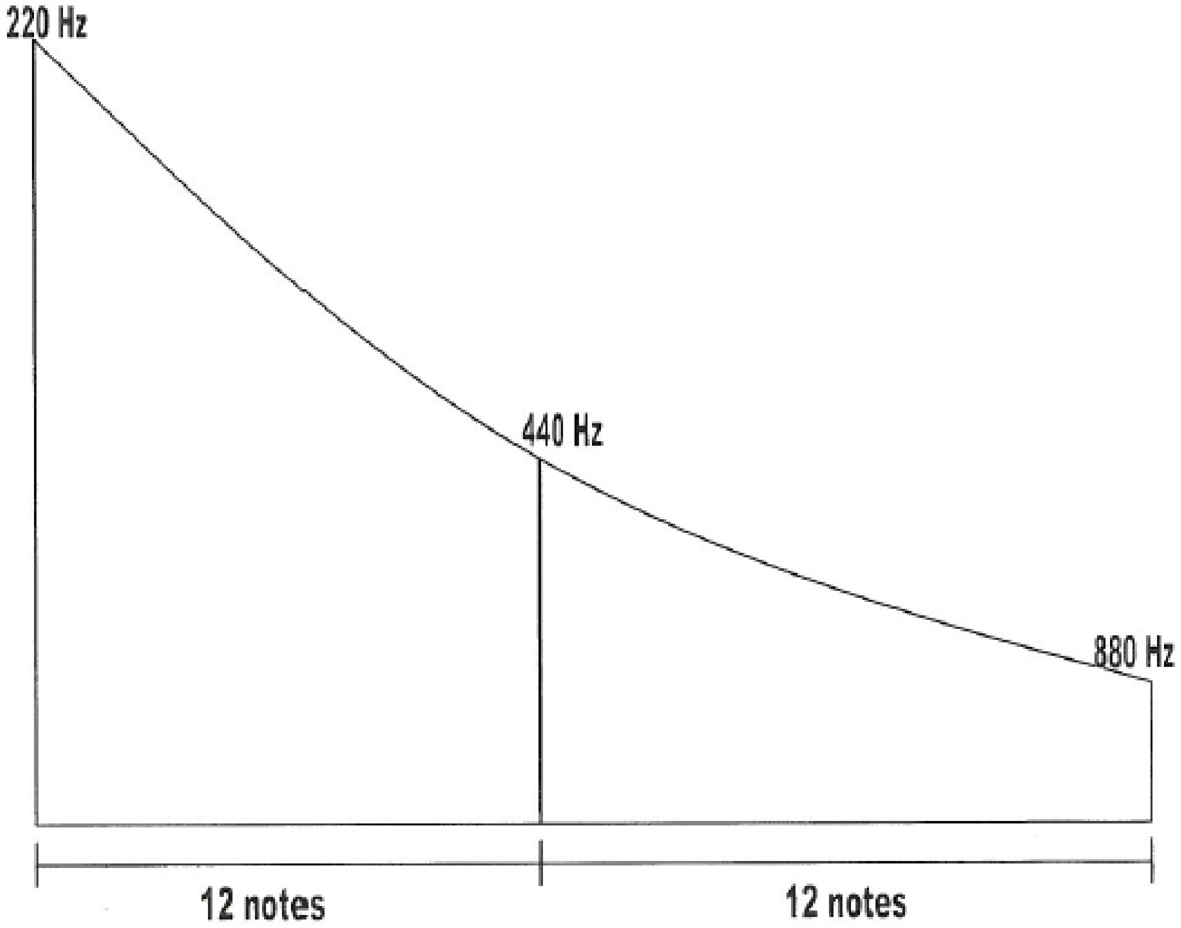
Room 407 was filled with curious minds, as we Algebra students pondered about the topic for the math fair. We grew anxious yet nervous, because this topic was both fascinating and new to us. Mr. Varum had constructed exemplary projects in the previous years, and how could we reach those standards with a topic so new to our young minds?

Much research was done about the frequencies of notes, and the direct relationships that it has with string lengths. Pythagoras had discovered the relationship between these frequencies, and how they related to string lengths. This idea that Pythagoras had presented to us gave us a strong understanding of what we wanted the basis of our project to be about. Eyes lit up as our minds had all interconnected. The idea of building a guitar had hit us in the head. We knew that this idea would challenge the minds of us individuals, but this was a challenge that we were willing to meet. Since a harp also had a direct relationship from string length to note frequencies, we had decided that developing a model for a harp would be an addition to the foundation that we had already built.

With an idea set, we had gotten our pencils and lifted our graph paper and got down to work. Our work consisted of creating a design for both the harp and the guitar. During that process, we used scale models and figured out the sectors and the areas of various shapes. In the making of these models we also used tangent, sine, and cosine ratios to precisely shape this guitar. We mathematically found arc lengths that would help in the outline of the guitar. Modeling and scaling the guitar and the harp was only one key component in the multiple steps in building a guitar. Exponential growth and decay was used to establish note frequencies, and string lengths. There was a certain ratio that would be used in both situations, that interconnects the notes with the strings. Proportional reasoning and combinations of metric and standard system of measurements were used in building the guitar. Much effort and teamwork was put into the making of this project. We have a more clear understanding of the mathematics behind music, and we are sure that we will bring this knowledge with us, and share with others our appreciation for fine music.

Formulas for Exponential Growth and Decay of String Lengths and Frequencies

According to Pythagoras if you have a string that is of a specific tension and halve it, the frequency of the smaller string will be double the frequency of the original.



If you attempt to connect these points you realize that this is an example of exponential growth and decay.

Since there are 12 notes in our scale a formula can be used to find the frequencies of these notes:

$$220(x)^{12} = 440$$

$$220x^{12} = 440$$

$$X^{12} = 2$$

$$\sqrt{x^{12}} = \sqrt{2}$$

$$X = 1.059463094$$

Guitar Design and Mathematics

Fret Placement from the Bridge of the Guitar

Nut - *650mm*

Fret 1 - 650*.943874327= *613.5183126 mm*

Fret 2 - 613.5183126*.943874327 = *579.0841844 mm*

Fret 3 - 579.0841844*.943874327 = *546.5826948 mm*

Fret 4 - 546.5826948*.943874327 = *515.9053732 mm*

Fret 5 - 515.9053732*.943874327 = *486.9498369 mm*

Fret 6 - 486.9498369*.943874327 = *459.6194496 mm*

Fret 7 - 459.6194496*.943874327 = *433.8229987 mm*

Fret 8 - 433.8229987*.943874327 = *409.4743909 mm*

Fret 9 - 409.4743909*.943874327 = *386.4923651 mm*

Fret 10 - 386.4923651*.943874327 = *364.800221 mm*

Fret 11 - 364.800221*.943874327 = *344.3255631 mm*

Fret 12 - 344.3255631*.943874327 = *325.0000592 mm*

Fret 13 - 325.0000592*.943874327 = *306.7592121 mm*

Fret 14 - 306.7592121*.943874327 = *289.5421449 mm*

Fret 15 - 289.5421449*.943874327 = *273.2913971 mm*

Fret 16 - 273.2913971*.943874327 = *257.9527336 mm*

Fret 17 - 257.9527336*.943874327 = *243.4749628 mm*

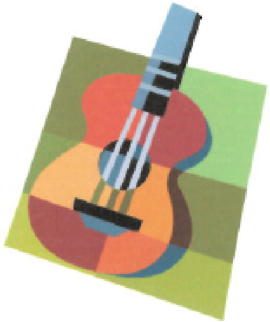
Fret 18 - 243.4749628*.943874327 = *229.8097666 mm*

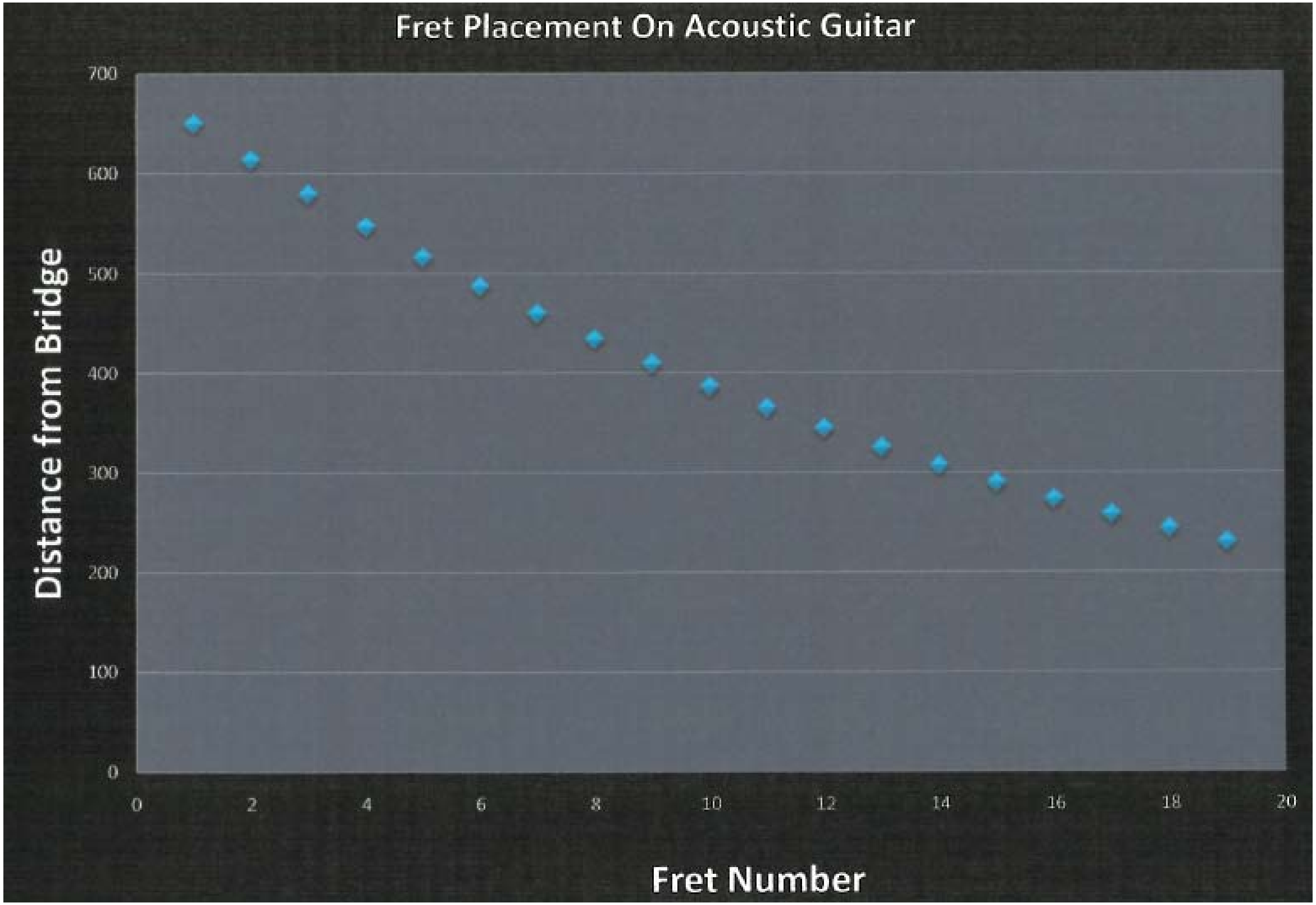
Rule of exponential decay used:

Y = 650(.943874327) ^X

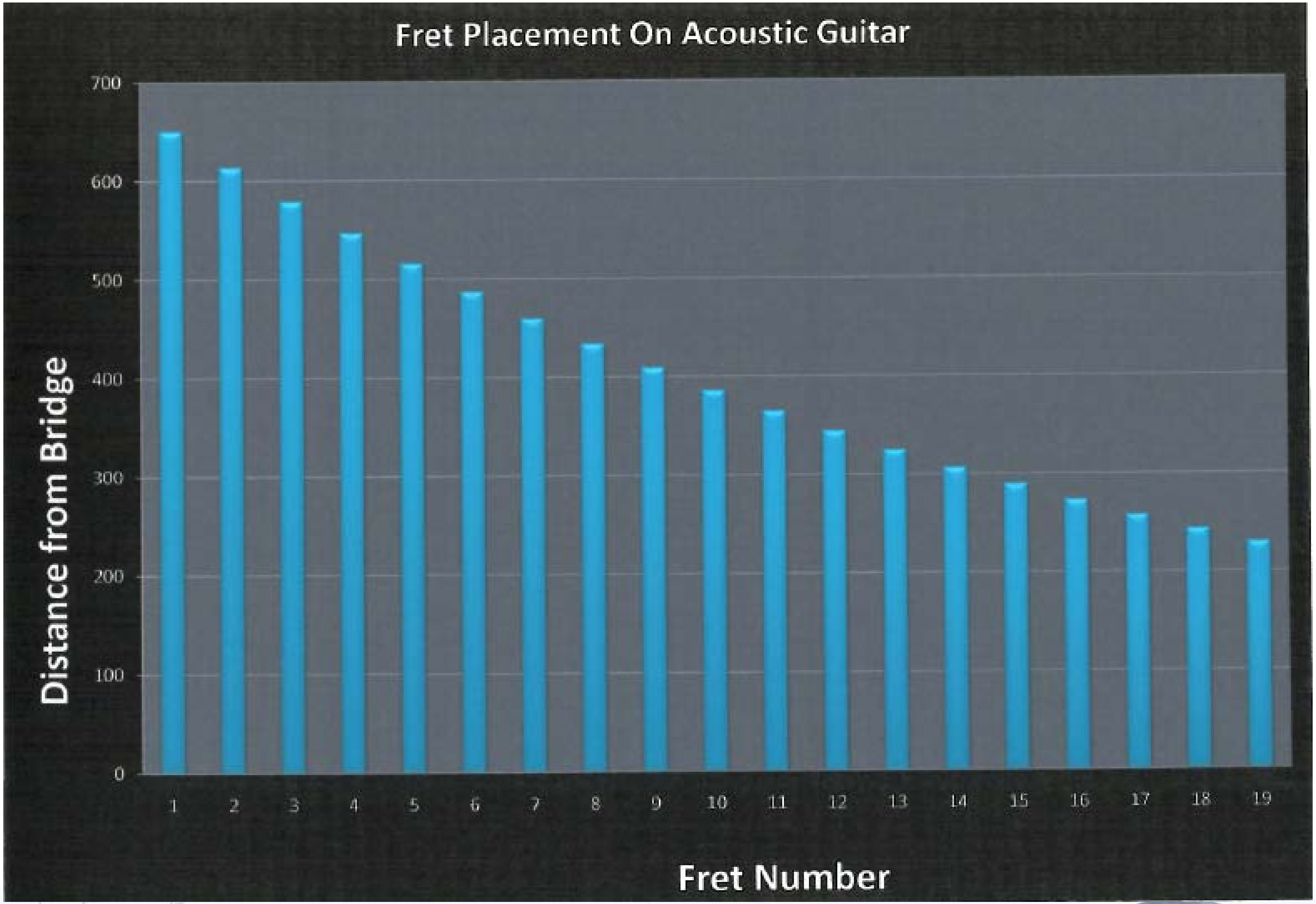
Note Frequencies for the Guitar

Notes	MIDI #	Frequencies
F2	41	6.875*2^((3+41)/12)=87.30705786
F#2	42	6.875*2^((3+42)/12)=92.49860568
G2	43	6.875*2^((3+43)/12)=97.998859
G#2	44	6.875*2^((3+44)/12)=103.8261744
A2	45	6.875*2^((3+45)/12)=110
A#2	46	6.875*2^((3+46)/12)=116.5409404
B2	47	6.875*2^((3+47)/12)=123.4708253
C3	48	6.875*2^((3+48)/12)=130.8127827
C#3	49	6.875*2^((3+49)/12)=138.5913155
D3	50	6.875*2^((3+50)/12)=146.832384
D#3	51	6.875*2^((3+51)/12)=155.5634919
E3	52	6.875*2^((3+52)/12)=164.8137785
F3	53	6.875*2^((3+53)/12)=174.6141157
F#3	54	6.875*2^((3+54)/12)=184.9972114
G3	55	6.875*2^((3+55)/12)=195.997718
G#3	56	6.875*2^((3+56)/12)=207.6523488
A3	57	6.875*2^((3+57)/12)=220
A#3	58	6.875*2^((3+58)/12)=233.0818808
B3	59	6.875*2^((3+59)/12)=246.9416506
C4	60	6.875*2^((3+60)/12)=261.6255653
C#4	61	6.875*2^((3+61)/12)=277.182631
D4	62	6.875*2^((3+62)/12)=293.6647679
D#4	63	6.875*2^((3+63)/12)=311.126987
E4	64	6.875*2^((3+64)/12)=329.6275569
F4	65	6.875*2^((3+65)/12)=349.2282314
F#4	66	6.875*2^((3+66)/12)=369.9944227
G4	67	6.875*2^((3+67)/12)=391.995436
G#4	68	6.875*2^((3+68)/12)=415.3046977
A4	69	6.875*2^((3+69)/12)=440
A#4	70	6.875*2^((3+70)/12)=466.1637615
B4	71	6.875*2^((3+71)/12)=493.8833013
C5	72	6.875*2^((3+72)/12)=523.2511306
C#5	73	6.875*2^((3+73)/12)=554.3645262
D5	74	6.875*2^((3+74)/12)=587.3295358
D#5	75	6.875*2^((3+75)/12)=622.2539674
E5	76	6.875*2^((3+76)/12)=659.2551138
F5	77	6.875*2^((3+77)/12)=698.4564659
F#5	78	6.875*2^((3+78)/12)=739.98888454
G5	79	6.875*2^((3+79)/12)=783.990872
G#5	80	6.875*2^((3+80)/12)=830.6093952
[illegible]	[illegible]	[illegible]



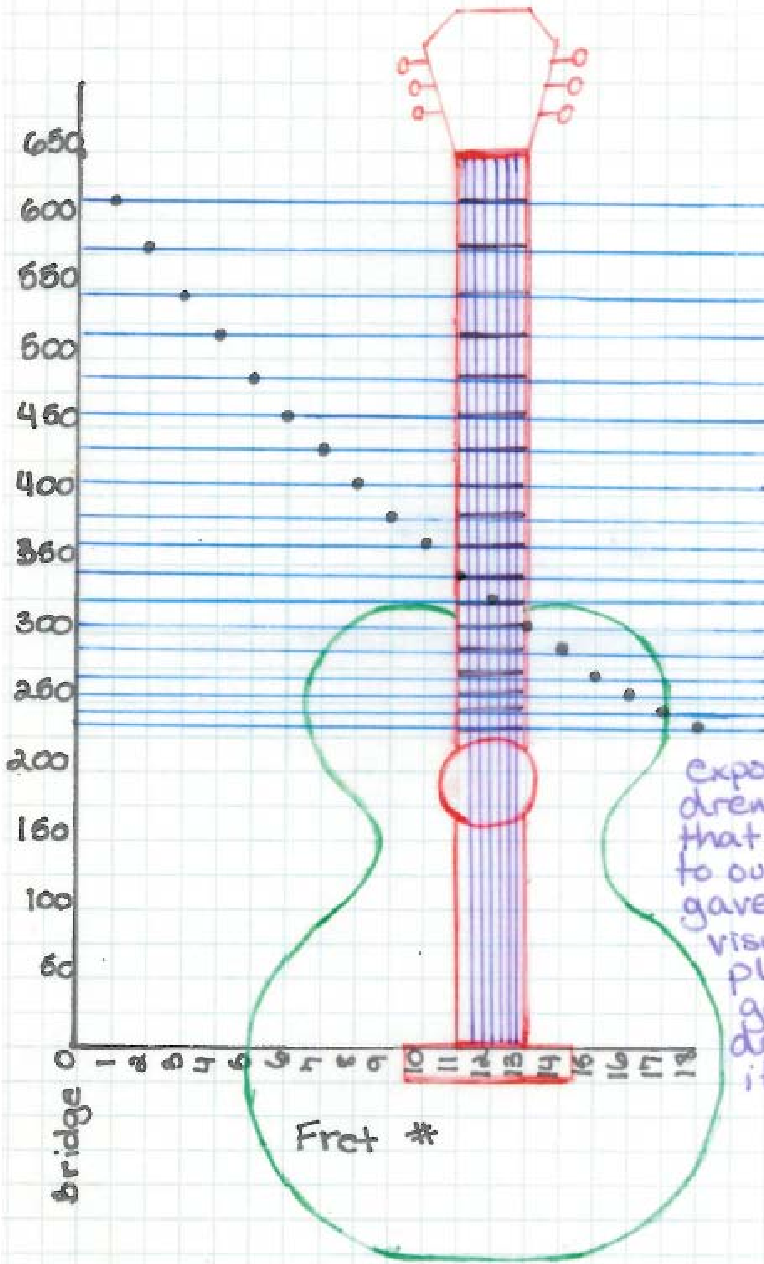


Fret Placement On Acoustic Guitar
Distance from Bridge
700
600
500
400
300
200
100
0
0
2
4
6
8
10
12
14
16
18
20
Fret Number



Fret Placement On Acoustic Guitar
Distance from Bridge
700
600
500
400
300
200
100
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Fret Number

Fret Placement on Acoustic Guitar



This is a visual representation of how we were able to calculate the fret positions on our guitar. As the note frequencies increased exponetially, the fret positioning decreased exponentially. We drew straight lines that were parallel to our x.axis. This gave us a clear visual of the fret placement. The guitar was then drawn to show its appearance.

Guitar Measurements

Total Width

1. Radius=5.5 in.
2. Diamter=11 in.

5.5+3+5.5= **14 in**

Radius of Smaller Circle

$a^2 + b^2 = c^2$

$4^2+6.5^2= C^2$

16+42.25=58.25

$C^2=58.25$

$\sqrt{58.25}= \sqrt{c^2}$

C=7.632168761 7.632168761-5.5= **2.132168761 in.**

Radius of Medium Circle

$a^2 + b^2 = c^2$

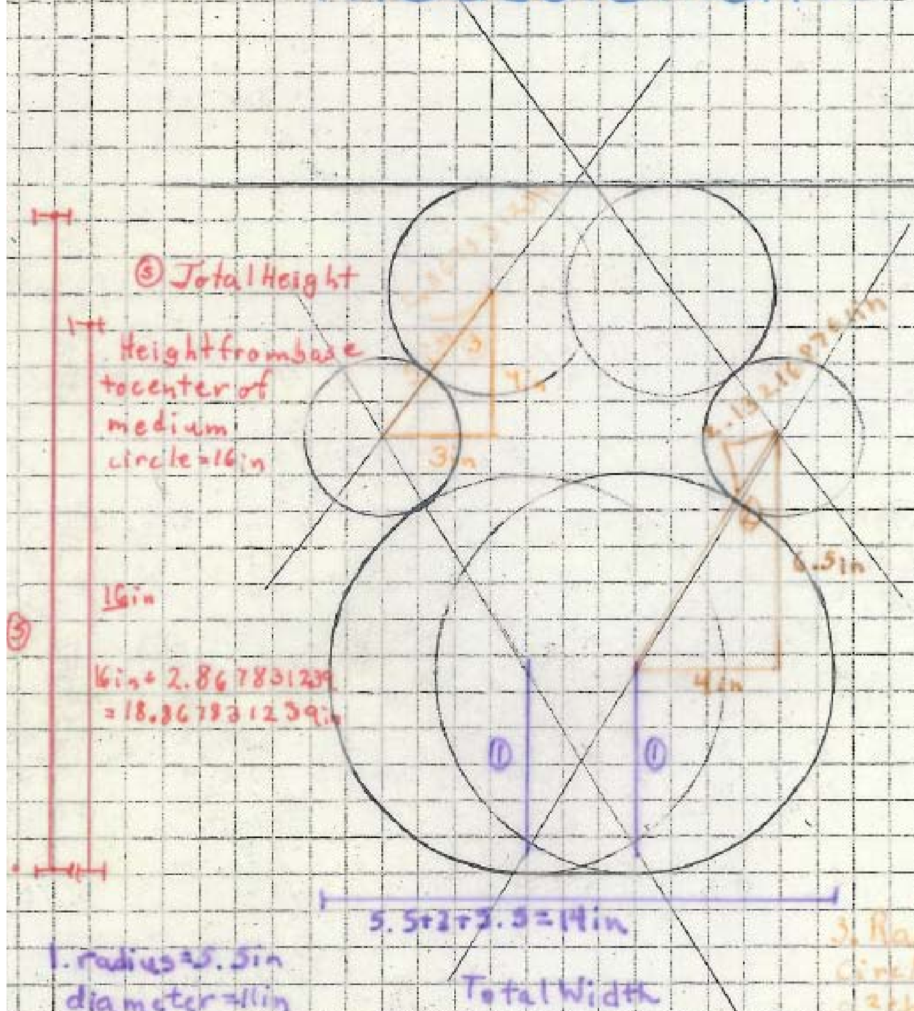
$3^2+4^2=c^2$

$25=c^2$

5=c — Distance from center of small circle to the center of the larger circle.

5-2.132168761= **2.867831139 in.** — **Radius of medium circle**

Guitar Measurements



1. radius = 5.5in
diameter = 11in

2. Radius of smaller circle
$a^2+b^2=c^2$
$4^2+6.5^2=c^2$
$16+42.25=58.25$
$c^2=58.25$
$\sqrt{58.25}=\sqrt{c^2}$
$c=7.632168761$

7.632168761 − 5.5
= 2.132168761in

3. Radius of Medium circle
$a^2+b^2=c^2$
$3^2+4^2=c^2$
$25=c^2$
$5=c$

Distance from center of small circle to edge of larger circle

Sector Angles of the Guitar

Central Angle of medium circle:

The black dotted triangle is congruent to the purple dotted triangle. The dotted purple triangle is similar to the 3-4-5 triangle, so it has equivalent angles.

180° - 36.86989765°= 143.1301301024°

Central Angle for large circle:

$\tan^{-1}(6.5/4) = X$

58.39249775=x

58.39249775+90= 148.3924978°

Central Angle of smallest circle:

$\tan^{-1}(3/4) = X$

36.86989765°= X

180-(36.86989765*2)

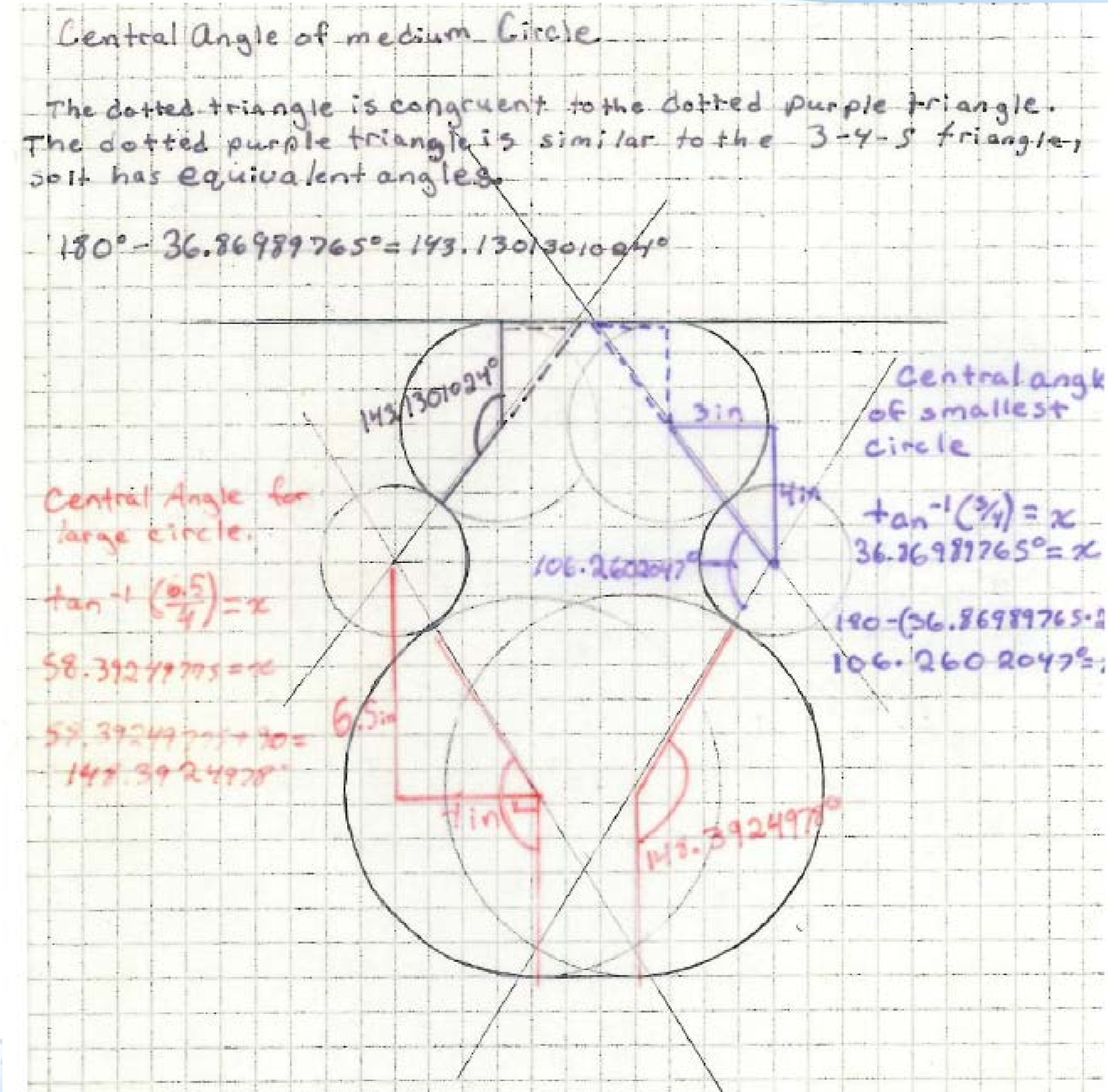
106.2602047°= X

Total Height

Height from base to center of medium circle= 16 in

16+2.867831239= **18.867831239 in.**

Sector Angles Of the Guitar



Perimeter of Guitar Sound Board

Circumference of large circle

2π (5.5)=34.55751919 in.

$$\frac{148.3924838°}{34.55751919} = \frac{X}{360}$$

X=14.2446572in.= ARC LENGTH

Circumference of medium circle

2π(2.867831239)=18.0191151 in.

$$\frac{143.1301024}{360} = \frac{X}{18.0191151}$$

X=7164104972 in. = ARC LENGTH

Purple Section

Circumference of small circle

2π(2.132168761)*13.39681143 in.

$$\frac{106.2602047}{360} = \frac{X}{13.39681143}$$

X=3.954299792 = ARC LENGTH

Total Perimeter:

2(14.2446572)+2(3.954299792)

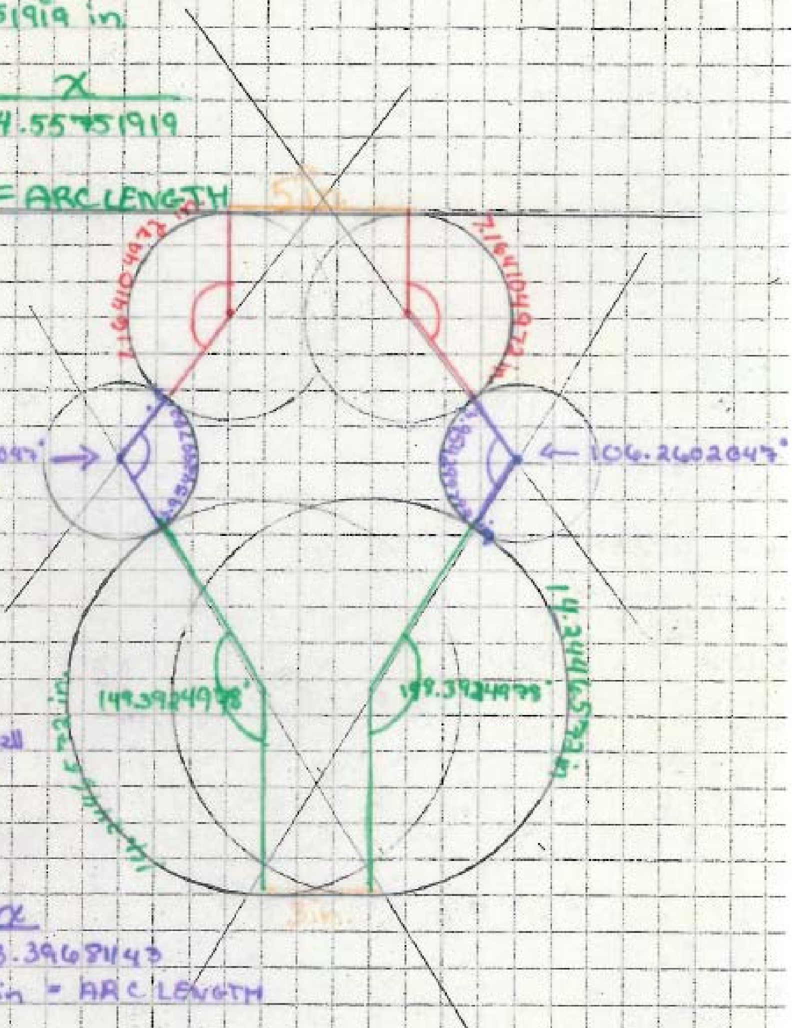
+2(7.164104972)+5+3=

58.72612393 in.

Circumference of large circle
$2\pi(5.5) = 34.55751919$ in

$$\frac{148.3924978^\circ}{360} = \frac{x}{34.55751919}$$

$x = 14.2446572$ in $=$ ARC LENGTH



Purple Section
Circumference of small circle
$2\pi(2.132168761) =$
13.39681143 in

$$\frac{106.2602047}{360} = \frac{x}{13.39681143}$$

$x = 3.954299792$ in $=$ ARC LENGTH

Circumference of medium circle

$2\pi(2.867831239) =$
18.0191151 in

$$\frac{143.1301024}{360} = \frac{x}{18.0191151}$$

$x = 7.164104972$ in $=$ ARC LENGTH

Total Perimeter

$2(14.2446572) + 2(3.954299792)$
$+ 2(7.164104972) + 5 + 3 =$

58.72612395 in

AREA OF THE FACE GUITAR SOUNDBOX

Green Section

Area of the Circle = πr^2

$\pi * 5.5^2 = 95.03317777 \text{ in}^2$

Sector Area

$$\frac{148.3924978^\circ}{360^\circ} = \frac{x}{95.03317777 \text{in}^2}$$

$148.3924978 * 95.03317777 \div 360 = 39.17280727 \text{in}^2$

Each Sector in Green is 39.17280727in^2

39.17280727x2=

Total area is 78.34561454 in^2

Area of the Red Section

Area of Sectors

$106.2602047^\circ - 58.39249775^\circ = 47.86770695^\circ$

$$\frac{47.86770695^\circ}{360^\circ} = \frac{x}{14.28213142 \text{in}^2}$$

$47.86770695 * 14.28213142 \div 360 = 1.899035783 \text{in}^2$

$1.899035783 * 2 = 3.798071566 \text{ in}^2$

The area of both red sectors combined is 3.798071566 in^2

Area of the Trapezoid = .5(h)(base1+base2)

$.5(4)(11+5) = 32 \text{in}^2$

32- 3.7980714566

28.20192843 in^2 = Area of the red section of the guitar.

Area of Purple section

Area of trapezoid ½*6.5(3+11) = 45.5 in^2
Tan^{-1} (6.5/4)=58.39249775°

Small Circle Area

$\pi (2.132168761)^2 = 14.28213142$

Sector Area

$$\frac{58.39249775}{360} = \frac{x}{14.28213142}$$

58.39249775*14.28213142÷ 360 = 2.316581463 in^2

2.316581463*2 = 4.633162833 in^2

45.5 in^2- 4.633162833 in^2 = **40.86683707 in^2= Area of purple section**

Area of Black Section

Area of black circle
$\pi(2.867831239)^2$= 25.837890 in^2

$$\frac{143.1301024}{360} = \frac{x}{25.8378906}$$

143.1301024x25.8378906/360=10.27272202 in^2

Area of each black section. 10.27272202x2=20.54544404 in^2
The total area of black section is 20.545444404 in^2

Area of Orange Section

$3x5.5=16.5 \text{ in}^2$

The total area of the Orange Section is 16.5 in^2

Area of Brown Section

$2.86783129x5=14.3391562 \text{ in}^2$

The total area of the brown section is 14.3391562 in^2

Face of The Guitar

$2(39.17280727)+40.86683707+28.20192843+2(10.27272202)+14.3391562+16.5=$

The total area of the guitar is **196.6381365 in^2**

Given Section

Area of Circle

$\pi r^2 = A$

$\pi 5.5^2 = 95.03317777 \text{ in}^2$

Sector Area

$\frac{148.3924978}{360°} = \frac{x}{95.03317777 \text{in}}$

$148.3924978 \times 95.03317777 \div 360 = 39.17280727$

$x = 39.17280727$ each

Area of green sectors

Area of Purple section

Area of trapezoid

$\frac{1}{2} \cdot 6.5(3+11) = 45.5 \text{in}^2$

$\tan^{-1}(\frac{6.5}{4}) = 58.39249775°$

Circle Area

$\pi \cdot (2.132168761)^2 =$

14.28213142

Sector Area

$\frac{58.39249775}{360} = \frac{x}{14.28213142}$

$58.39249775 \times 14.28213142 \div 360 =$

2.316581463in^2

$2.316581463 \times 2 =$

4.633162833in^2

$45.5 \text{in}^2 - 4.633162833 \text{in}^2 =$

40.86683707in^2 = Area of Purple section

Area of Red trapezoid

Area of Sectors

$106.26020477° - 58.39249775$

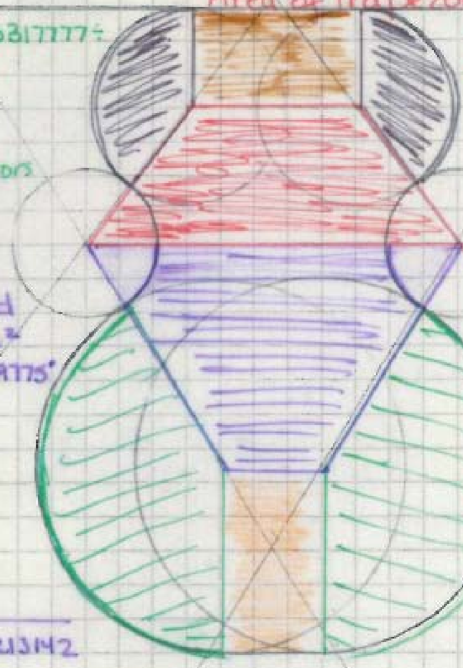
$47.86770695°$

$\frac{47.86770695}{360} = \frac{x}{14.28213142}$

$47.86770695 \times 14.28213142 \div 360 =$

$1.899035783 \times 2 = 3.798071566 \text{in}^2$

Area of Trapezoid $\frac{1}{2}(4)(11+5) = 32 \text{in}^2$



$32 \text{in}^2 - 3.798071566 \text{in}^2$

28.20192843in^2

Area of Red section

Area of Black section

Area of Circle

$\pi(2.867831239)^2$ 2

25.8378906in^2

$\frac{143.1301024}{360} = \frac{x}{25.83789}$

$143.1301024 \times 25.83789$

$\div 360 = 10.27272202$

Area of each Black section

Area of Orange section

$3 \times 5.5 = 16.5 \text{in}^2$

Area of Brown section

$2.867831239 \times 5 = 14.3391562 \text{in}^2$

Total Area

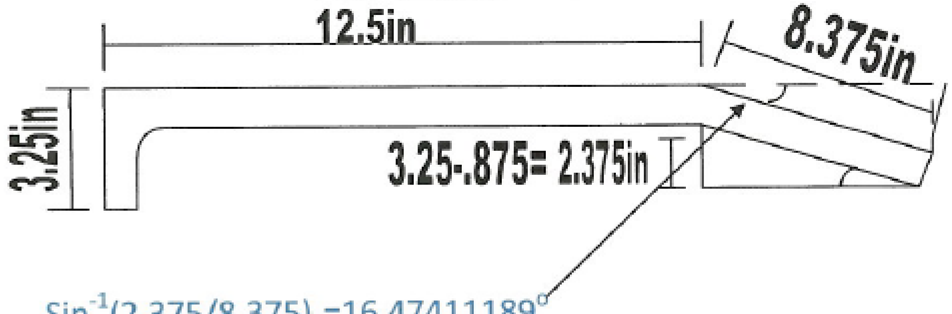
$2(39.17280727) + 40.86683707 +$

$28.20192843 + 2(10.27272202) +$

$14.33915620 + 16.5 =$

196.6381365in^2

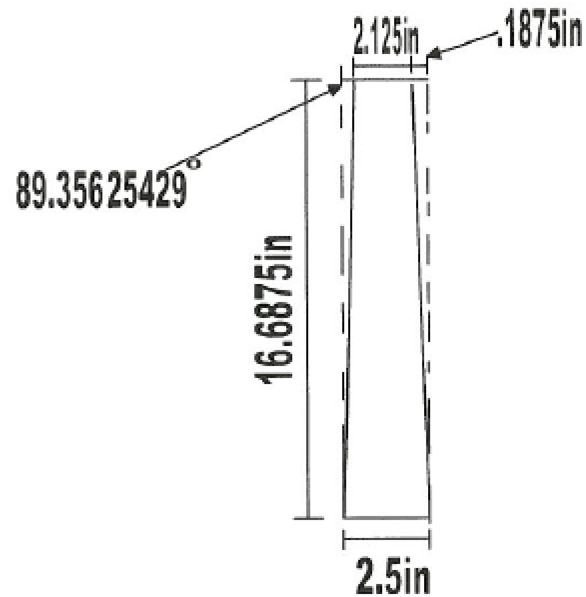
NECK MEASUREMENTS



$\text{Sin}^{-1}(2.375/8.375) = 16.47411189^{\circ}$

The angle of the handle is 16.47411189°.

FINGER BOARD MEASUREMENTS



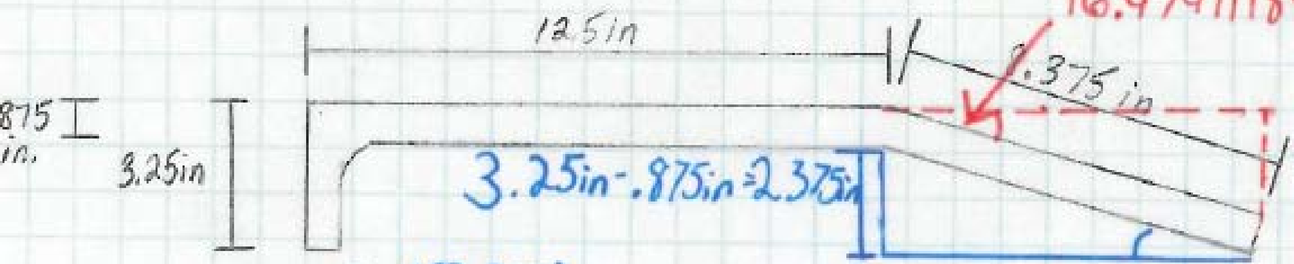
$2.5 - 2.125 = .375/2 = .1875$

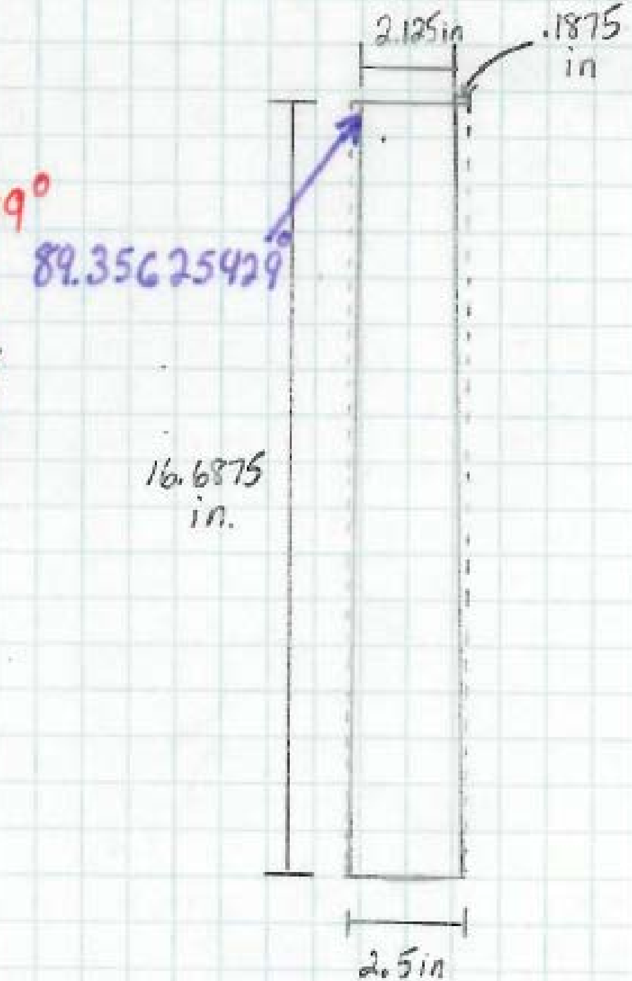
$\text{Tan}^{-1}(16.6875/.1875) = 89.35625429$

The angle of the taper is 89.35625429°

FINGER BOARD MEASUREMENTS

NECK MEASUREMENTS





$\sin^{-1}\left(\frac{2.375}{8.375}\right) = 16.47411189°$

The angle of the handle is 16.47411189°.

$2.5in - 2.125in = \frac{.375in}{2} = .1875$

$\tan^{-1}(16.6875/0.1875) = 89.35625429°$