Math Connections to Earth and Space Science

Foundations and Mathematical Application of the Water Rocket

Students Utilize

- 2-Liter Bottle Rocket Launcher will help students to explore the science of rocketry.
- Students build &propel 2-liter bottle rockets to altitudes up to 400 feet with just water and air pressure.

Students design and make a rocket that must travel a specific height and distance and land in a certain location.

Students:

- Consider features to incorporate into their designs
- Decide what materials to use
- Analyze strengths and weaknesses of their rocket
- Design a parachute and launch their rocket!

Curriculum Links include:

Science:

Practical and Inquiry Skills; Forces

Technology:

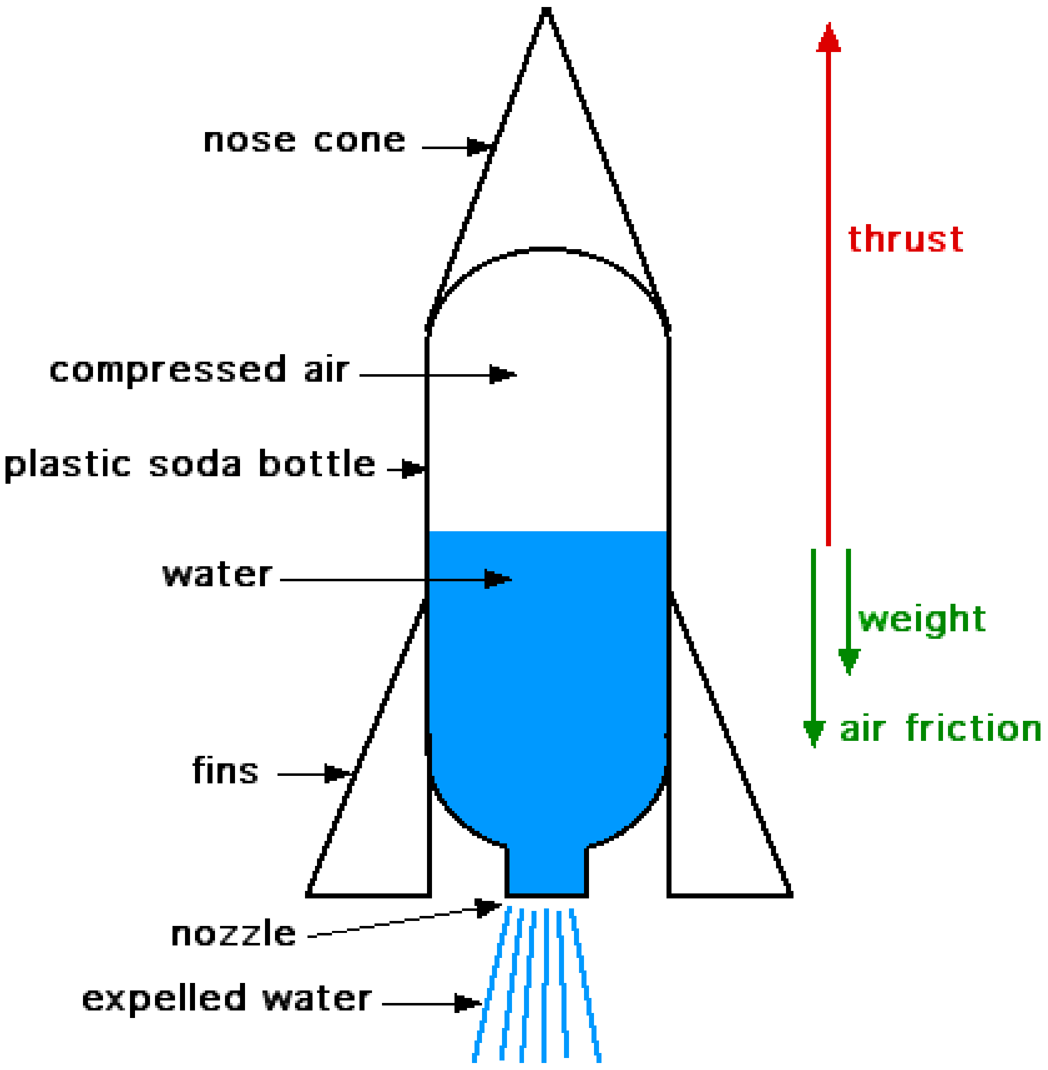
Developing, Planning & Communicating Ideas

Math:

Processes and Applications including

- Geometrical Reasoning
- Measures and mensuration

Cross section of a typical water rocket illustrates the principle of operation



How High Did it go?

When launching rockets, we like to know how high they go. Two ways to measure this:

(1) Put an expensive altimeter on the rocket and hope it survives the next crash, or
(2) Use trigonometric data from one or more remote observers with ***inclinometers*** and ***compasses***.

Here, I propose a 3rd method requiring only one remote observer with a stopwatch.

How High Did it go?

The new method to calculate apogee *depends on* a mathematical model, described below, for rocket flight following the ***thrust phase***.

Model development uses calculus-based classical physics, knowledge of this is *not* required to use the results of the model

The Thrust Phase

One of the distinguishing features of water rockets is that the **Thrust Phase** of the flight takes place quickly.

- Typically it takes <0.1 second to **expel water**, & perhaps 0.05 second to **expel high-pressure air**.
- Once thrust is finished, a certain amount of **TIME** will have elapsed since “lift off” and the rocket will be at a particular **HEIGHT** above the ground.

The Coasting Phase

For the remainder of the flight after the **Thrust Phase**, Newton‘s law give the following equation for rocket motion, the **Coasting Phase**:

$$m\,\boldsymbol{a} = \boldsymbol{F}_{gravity} + \boldsymbol{F}_{drag}$$

m =mass of the (empty) rocket
a =its acceleration
Fs =forces on the rocket.

The Coasting Phase

F_{drag} changes sign during flight:

During ascent, it is negative—downward acting

During descent, it is positive—upward acting

The Science of Speed: Drag & Drafting

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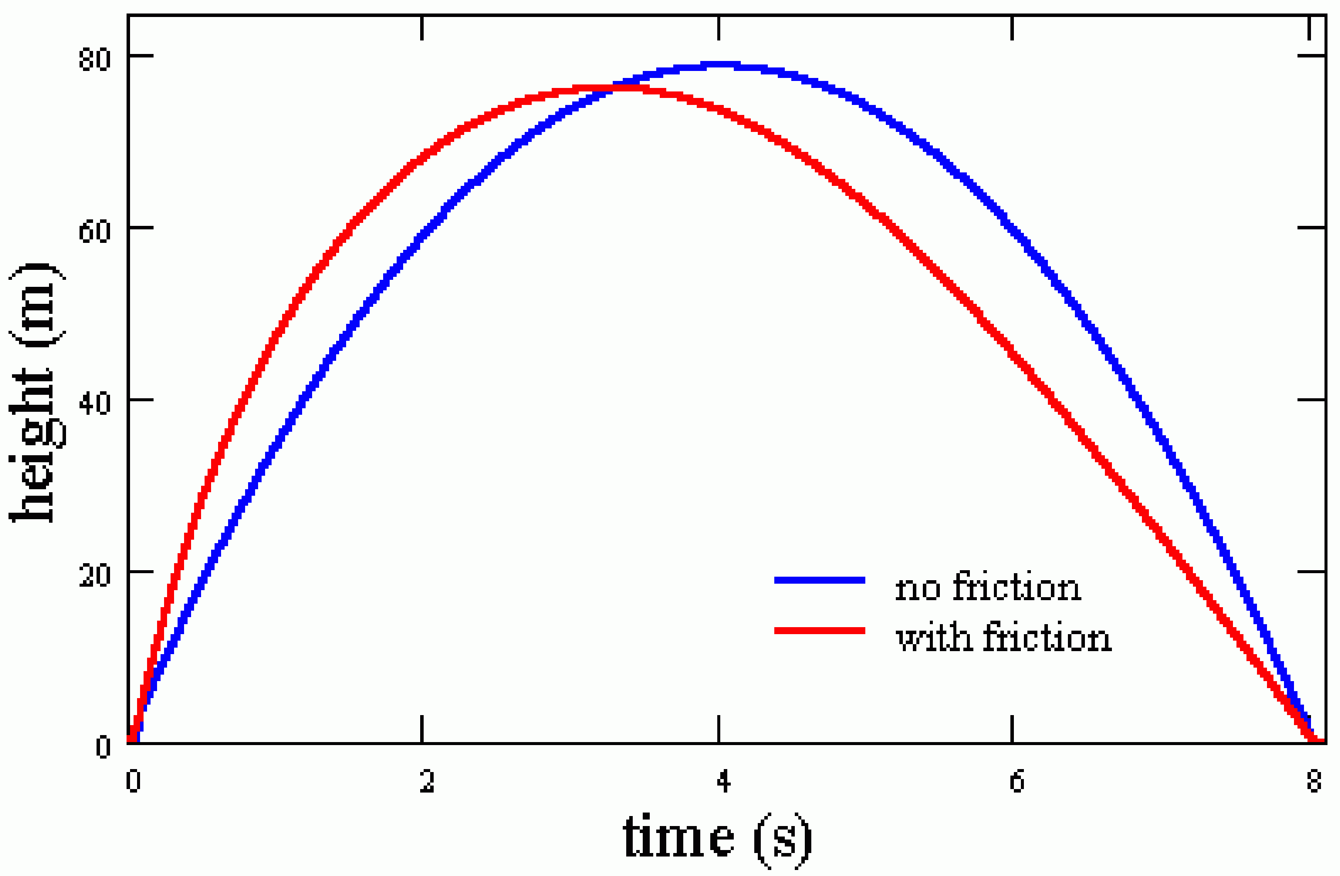
Drag & Drafting: Engine power is constrained at superspeedways like Daytona and Talladega, so teams use aerodynamics to gain an advantage. Teams adjust their cars to minimize drag, but then it's up to the drivers to find 'the draft' and to trust the drivers behind them to literally "bump" them into Victory Lane.

The Simplest Correlation



Middle School

Obviously for Middle School students …focus on:

- Volume $V = r^2h$

  r = radius of cylinder

  h =height or length of cylinder

- Altitude = Base x Tangent
- Speed=Distance /Time [i.e, miles/hour]
- Thrust =exhaust velocity w/respect to rocket

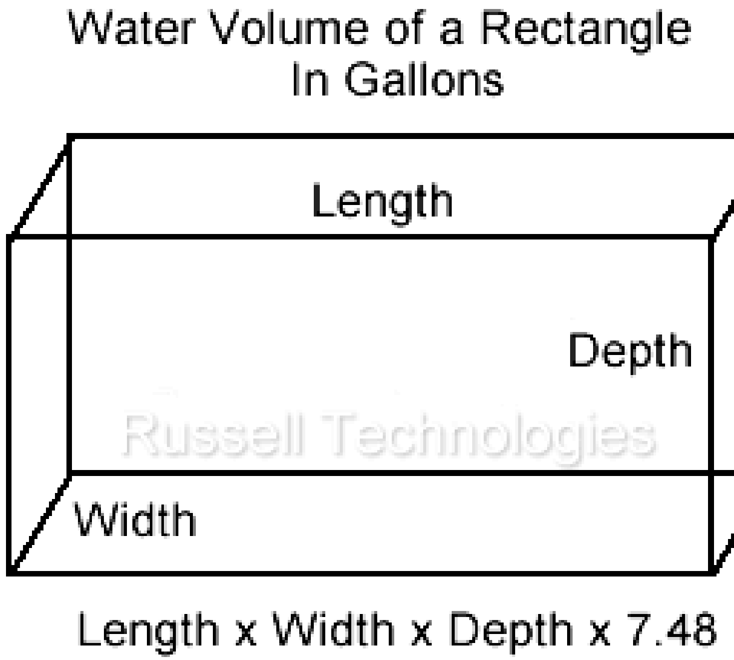
Volume

All water features can be broken down into one or more of these shape categories.

Volume Calculations for each shape is listed for you below.

For "odd-shaped" water features, break down water features into smaller categories that fit into the shapes listed below – Then, add them all together for a close approximation of volume and gallons.

Volume

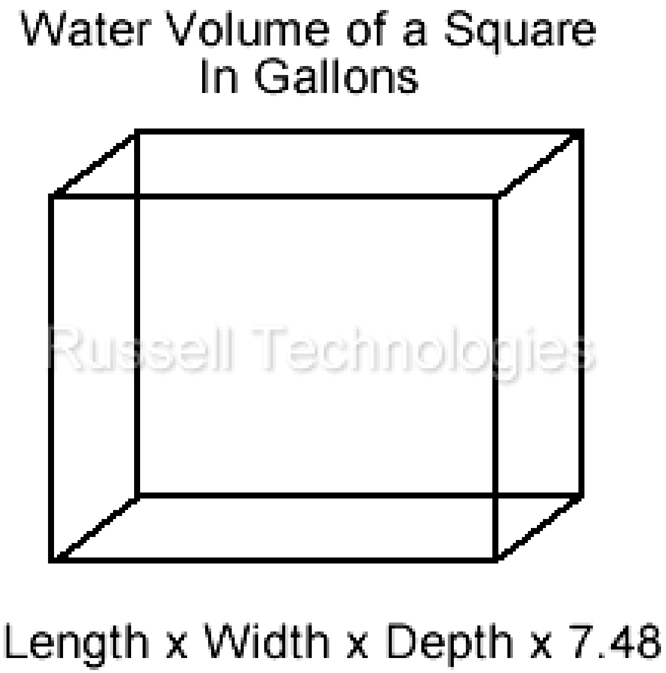


Volume of a Rectangle
Length x Width x Depth
= Cubic Feet.

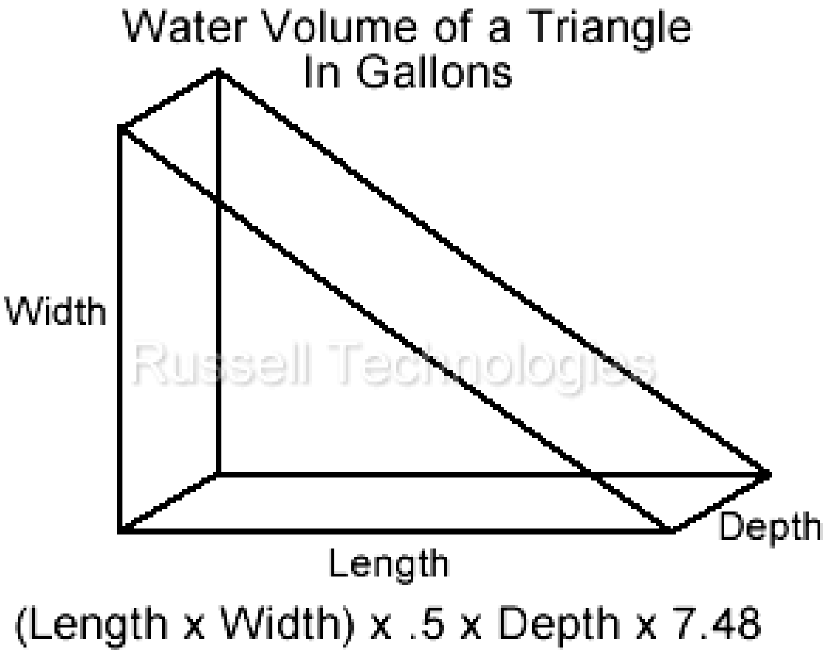
Cubic Feet x (7.48) =
Total Gallons.

Volume

Volume of a Square
Length x Width x Height = Cubic Feet.
Cubic Feet x (7.48) = Total Gallons.



Volume



Volume of a Triangle

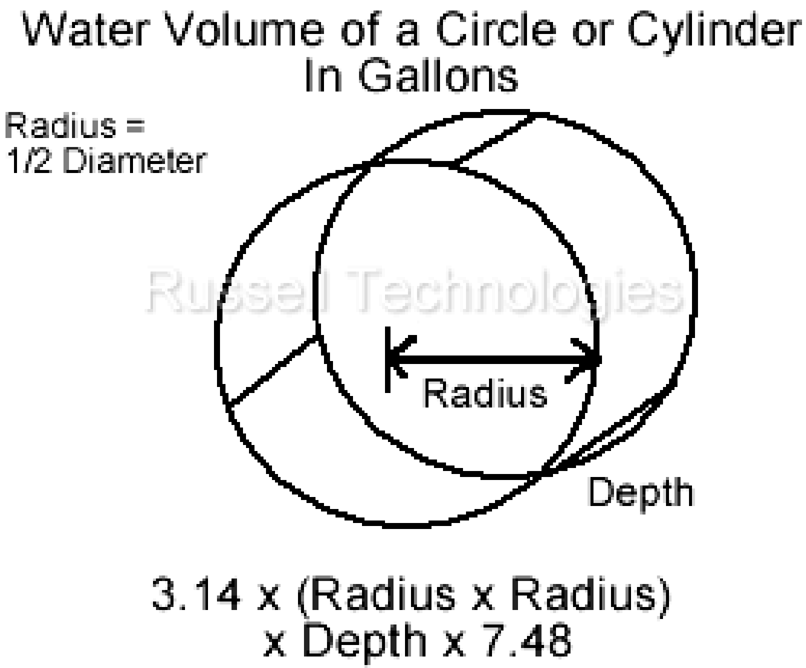
(Length x Width)(.5)(Height)
= Cubic Feet.

Cubic Feet x (7.48) =
Total Gallons

Volume

Volume of a Circle or Cylinder

π $\mathbf{r^2}$ x height= Cubic Feet
Cubic Feet x (7.48) = Total Gallons.



* radius = ½ diameter

Altitude

- Elevation especially above sea level or above the Earth's surface

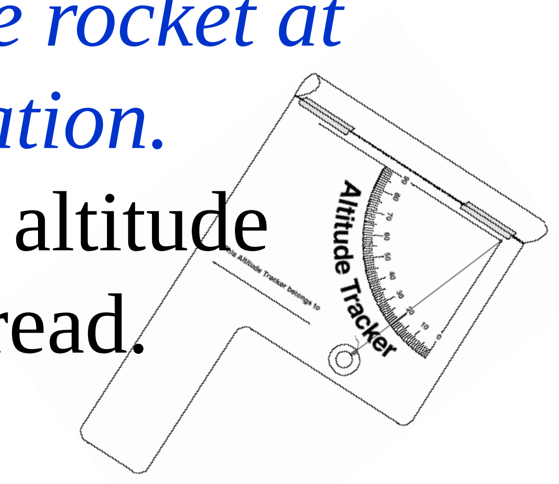
- The perpendicular distance from the base of a geometric figure to the opposite vertex (or side if parallel)

- Elevation: angular distance above the horizon (especially of a celestial object)

Altitude

Determining the altitude a rocket reaches in flight is a team activity. While one group of students prepares and launches a rocket, a second group measures the altitude the rocket reaches *by estimating the angle of the rocket at its highest point from the tracking station.*

The angle is then input into the altitude tracker calculator and the altitude is read.



Altitude

- Set up a tracking station location a short distance away from the rocket launch site.
- Depending upon the expected altitude of the rocket--tracking station should be **5, 15, or 30m** away.

Generally, a 5-meter distance is sufficient for paper rockets and antacid-power rockets. A 15-meter distance is sufficient for bottle rockets, and a 30-meter distance is sufficient for model rockets.

Altitude

- As a rocket launches, the person doing the tracking follows the flight with the sighting tube on the tracker.
- The tracker should be held like a pistol and kept at the same level as the rocket when it is launched.
- Continue to aim the tracker at the highest point the rocket reached in the sky.
- Have a second student read the angle the thread or string makes with the quadrant protractor.
- **Record the angle.**

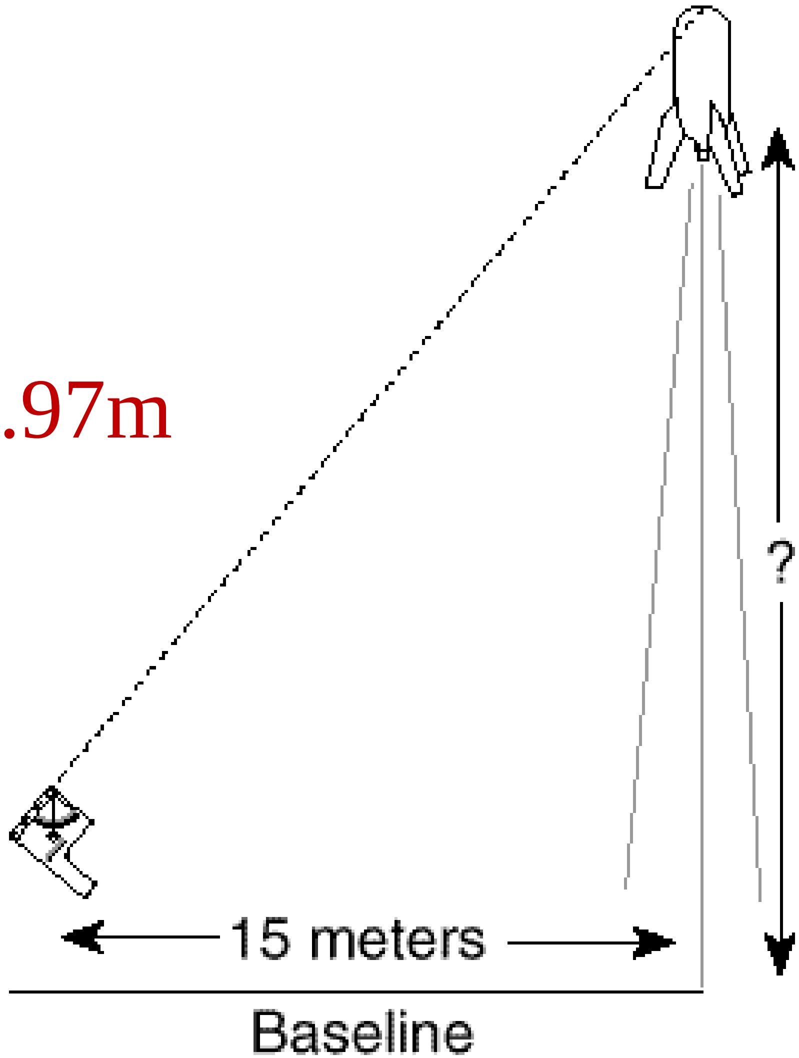
Altitude

- Once you determine the angle of the rocket, use the following equation to calculate altitude of the rocket: *Altitude* = tan*A* × *baseline*
- Use a calculator with trigonometry functions to solve the problem or refer to the tangent table.

For example, if measured angle is 28° and the baseline is 15m, the altitude is 7.97m

Altitude

$$Altitude = \tan\ 28° \times 15\text{m}$$
$$Altitude = 0.5317 \times 15\text{m} = 7.97\text{m}$$



SPEED

Speed = distance /time

Speed (velocity)= $\frac{\text{change in distance}}{\text{change in time}}$

or V = D/T

V = speed, D = distance &T = time

$$\frac{dv}{dt}:$$

Time Rate of Momentum Change/THRUST

A rocket is propelled forward by a (thrust) force equal in magnitude, but opposite in direction, to **the time-rate of momentum change of the exhaust gas accelerated from the combustion chamber through the rocket engine nozzle.** This is the exhaust velocity, **V**, (w/respect to the rocket) multiplied by the time-rate at which the mass is expelled, or:

$$\mathbf{T} = \frac{dm}{dt}\mathbf{v}$$

THRUST

- **T** = thrust generated (force)
- $\frac{dm}{dt}$ is the rate of change of mass with respect to time (mass flow rate of exhaust);
- **V** = speed of the exhaust **gases** measured relative to the rocket.

For vertical launch of a rocket the initial thrust must be more than the weight.

Water Rocket Analysis

Unfortunately the compressed air pressure P is not a constant during the thrust phase, but varies in a nonlinear manner with the expanding volume of the compressed air.

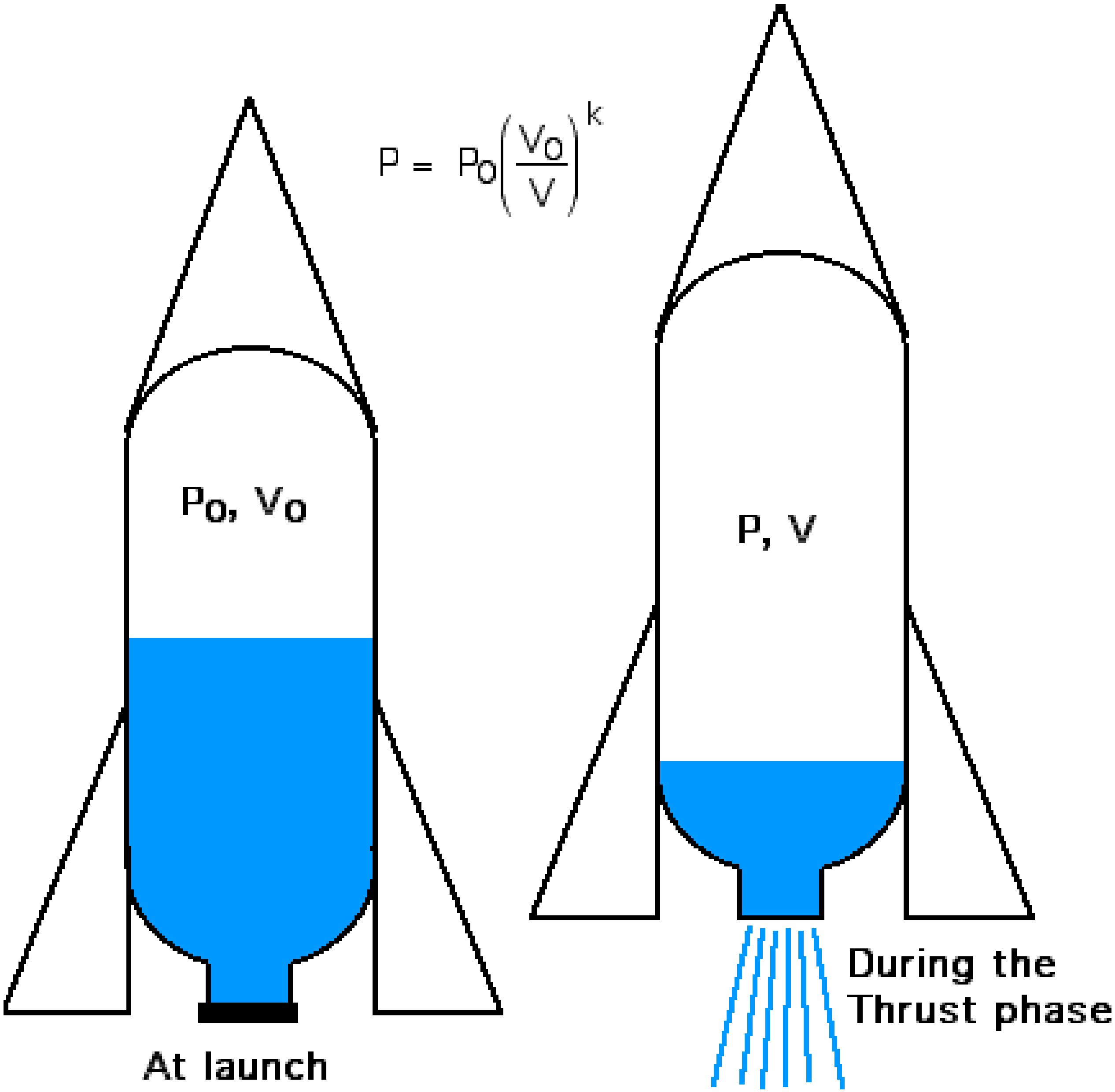
This is the main reason for the extremely complex relations resulting from this analysis.

Adiabatic Expansion

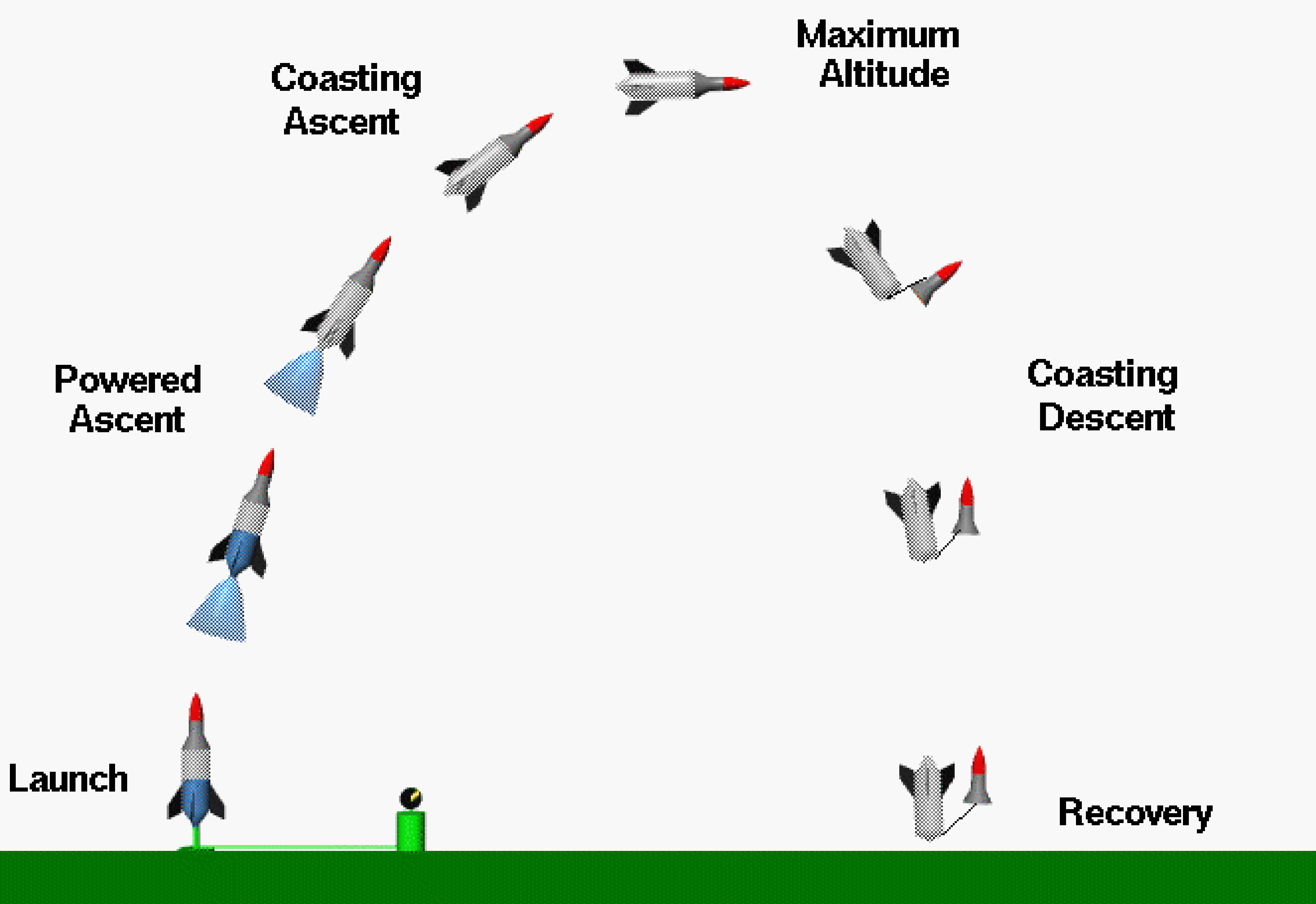
As the water escapes, the air volume increases, causing a decrease in pressure and a corresponding decrease in thrust.

We consider this process to be adiabatic (no transfer of heat during the split-second expansion process), which allows us to relate the time variation of the pressure to that of the volume.

Adiabatic Expansion

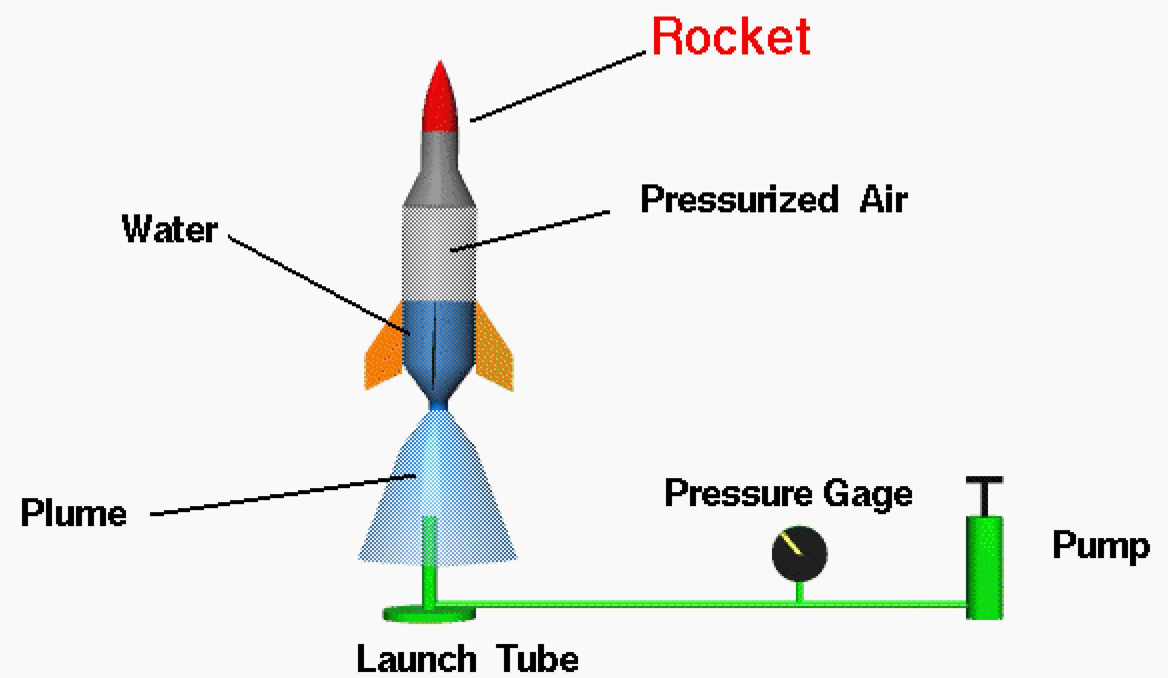


Flight of a Water Rocket

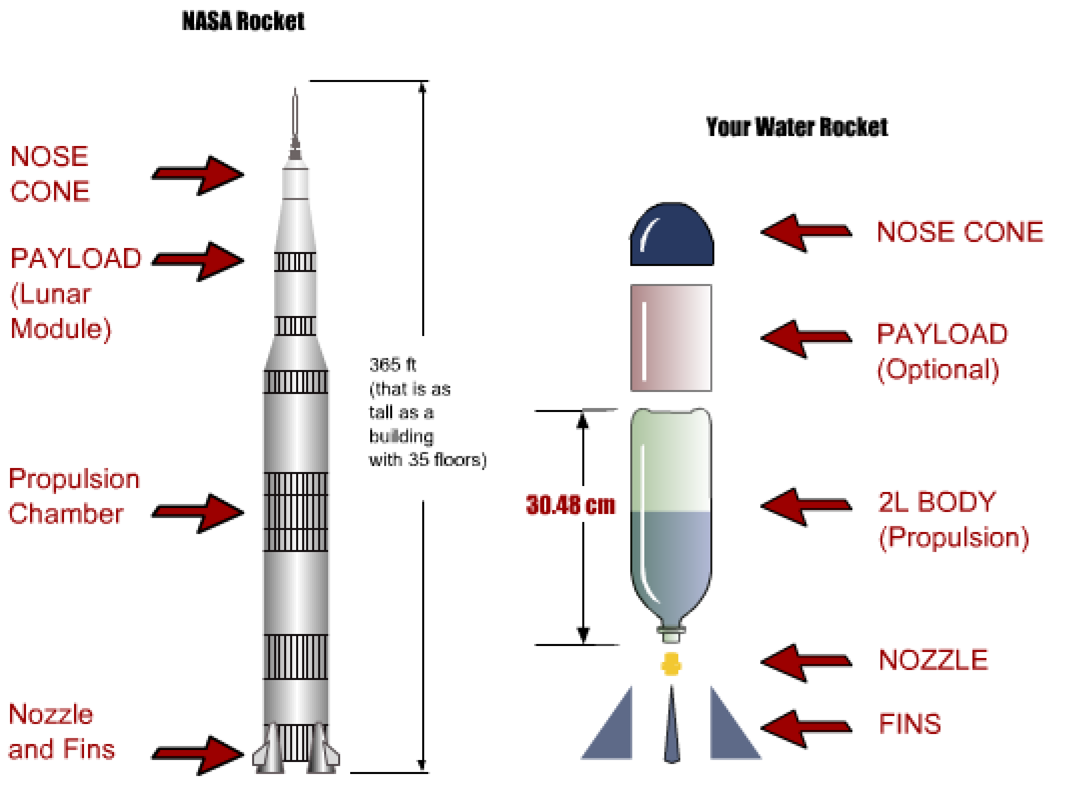


Flying model rockets is a relatively safe and inexpensive way for students to learn the basics of forces and the response of a vehicle to external forces. A model rocket is subjected to four forces in flight; weight, thrust, and the aerodynamic forces, **lift and drag**. There are many different types of model rockets. One of the first and simplest type of rocket that a student encounters is the bottle, or **water rocket**. The water rocket system consists of two main parts, the launcher and the rocket.

Water Rocket



Comparison to NASA Rocket



Resources

PITSCO EDUCATION

Advanced Bottle Rocket Construction Fixture (W24660)

$39.00
More Details

Dr. Zoon Stratoblaster Video (DVD) (W59667)

$24.95
More Details

Rocket Fin Holder (for 20 oz bottle) (W31728)

$14.95
More Details

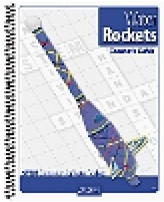
Water Rockets – Getting Started Package (W35563)

$285.00
More Details

StratoBlaster Bottle Rocket Kit with CO_2 Cartridge (W13235)

$8.50
More Details

Water Rockets Teacher's Guide (W59462)



$24.95
More Details

Resources

"Two-Liter Pop Bottle Rockets may well be the GREATEST PHYSICAL SCIENCE TEACHING TOOL EVER CREATED!!" Middle grades students can manipulate and control variables, see their hypotheses verified or refuted, and graph their findings. High school students experience the nature of science at its best. They can document their abilities with the following concepts: inertia, gravity, air resistance, Newton's laws of motion, acceleration, relationships between work and energy or impulse and momentum, projectile motion, freefall calculations, internal and external ballistics, and the practice of true engineering.